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The use of a voltage coefficient in defining the magnitude of the non-linearity of fixed resistors has serious deficiencies and an alternative method is proposed, involving the measurement of third harmonic distortion. In a simple case, a third harmonic output is produced, proportional to the cube of an applied fundamental voltage and on this basis a new unit entitled 'Third Harmonic Index' (t.h.i.) is derived. An explanation is suggested for the origin of non-linearity and a reason given for its correlation with current noise. Typical values of t.h.i. are quoted for film resistors over a wide range of values and future applications include its use in the evaluation of basic resistor performance, for test specification purposes and as a method of screening to ensure very high reliability.

MANY electronic components in the form of two-terminal devices are required to exhibit a perfectly linear relationship between two dependent variables. Examples of this include resistors (current and voltage), capacitors (charge and voltage) and inductors (rate of change of current and voltage). Any divergence from such a strictly linear relationship is usually regarded as a fault condition and is therefore to be avoided.

There are generally two ways in which 'non-linearity' makes itself apparent in supposedly linear components. In the first place at some high level of one of the variables there can be a sudden onset of non-linearity such as would occur in a 'breakdown' condition. Secondly, there may be a small but consistent deviation from strict linearity present at all levels of use as, for example, occurs when a resistor exhibits some slight departure from Ohm's law. It is this latter type of imperfection which will be discussed.

In the case of fixed resistors non-linearity can be described in terms of an apparent voltage coefficient. It is not surprising that, as a defect, this term has often been quoted in a context together with other non-perfect parameters such as temperature coefficient, instability or drift, current noise etc. Indeed, at a time when component manufacturers are striving hard to increase the reliability of their products is it appropriate that the maximum possible attention should be directed towards all aspects in which the component falls short of perfect or ideal behaviour.

Not all types of fixed resistor exhibit non-linearity to a measurable degree. Resistors which utilize a metallic conductor, say in the form of a wire, exhibit neither ohmic non-linearity nor current noise over the range of measurement permitted by available apparatus. In contrast, resistors utilizing a semiconducting material either in a solid form of construction or as a film on an insulating former do manifest both non-linearity and current noise. This is an indication that both of these phenomena, whose apparent relationship is discussed later, are related to the nature of the resistive material concerned.

The non-linearity of the current/voltage characteristic in any of the usual types of fixed resistor is extremely small and cannot be detected in a simple linear graphical presentation of current versus voltage. The effect can be observed using sensitive apparatus and is sometimes assessed by a direct measurement of the 'voltage coefficient' of resistance. This approach suffers from certain disadvantages. The normal procedure involves the rapid measurement of resistance at two levels of voltage. The method cannot be applied to film resistors due to the inevitable heating effect of the element and the variation in resistance which results from the effect of the temperature coefficient. An improved system involving the application of power for only 0·2sec was utilized in a commercial apparatus for the measurement of voltage coefficient. This, too, was found to be more sensitive to the temperature coefficient of film resistors than to their non-linearity.

A further serious disadvantage of a 'voltage coefficient'

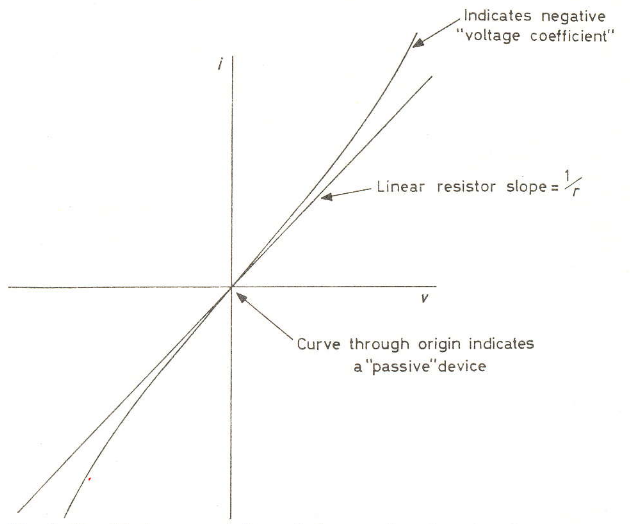

Fig. 1 Graphical representation of the non-linearity of a fixed resistor

relates to the very considerable dependence of this parameter on the level of applied voltage at which it is measured. If the relationship between current and voltage departs from perfect linearity, then it is hardly to be expected that the 'resistance' (derived from the slope of the i/v curve) will, in turn, vary linearly with the voltage and provide a 'voltage coefficient' which is constant at all levels of applied voltage.

An alternative indication of the non-linearity of a resistor may be obtained from the measurement of the amplitude of harmonics generated in the resistor when a pure fundamental sine wave signal is applied. Such harmonics can be detected by the use of sensitive apparatus[1] and commercial equipment has been developed to measure the ratio of the magnitude of the third harmonic component to that of the applied fundamental signal.

The small amount of non-linearity which is usually found in fixed resistors can be represented by a mathematical expression of the following type:

$$i = \alpha v + \beta v^3 + \gamma v^5 + \delta v^7 + \ldots \qquad \ldots\ldots\ldots (1)$$

This expression utilizes only the odd powers of a polynomial series and represents a device which is 'symmetrical' and 'passive' in the electrical sense (Fig. 1). The deviation from perfect linearity is usually very small

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and coefficients of increasing order in the above expression rapidly decrease in magnitude. It is found that, in all normal cases, the voltage coefficient is negative and this implies that the sign of β representing the third order coefficient is negative. The effect of a device having this characteristic on a pure a.c. waveform will now be considered. Due to the symmetry of the above expression, the output will contain a number of *odd* harmonics. An indication of the magnitude and phase relationship of the harmonics is given by the Fourier series derived in Table 1. It is, of course, most important to distinguish between a *coefficient* which represents the magnitude of the terms of increasing order in the polynomial expression representing current/voltage characterestic and the *amplitude* of the harmonics of ascending order. Nevertheless, an interesting practical relatonship between the two does emerge. In the first place the expression for the amplitude of the n^{th} harmonic includes factors relating only to the n^{th} *and higher order* coefficients of the original expression. Furthermore, as the n^{th} coefficient is likely to be considerably greater than the next higher coefficient it is likely that the magnitude of the n^{th} harmonic will largely be governed by the magnitude of the n^{th} order coefficient. For example, Millard[1] showed that, with an $\frac{1}{8}$W carbon film resistor at all levels up to 0·75W, the third harmonic could be attributed solely to the third order term of the original expression. The equations suggest that such a simplification will not be valid if the applied voltage is large, as this will augment the effect of the higher order coefficients.

If, for simplification, it is assumed that fifth and higher order curvature is negligible, then the current/voltage relationship is represented by a cubic expression. It is then found that the amplitude of the third harmonic voltage is proportional to the third power of the amplitude of the fundamental applied waveform. At low powers it has been found that this relationship holds for fixed film resistors and solid composition resistors[2]. At their normal ratings the latter produce third harmonic voltages dependent more nearly on the square of the applied fundamental voltage. Film resistors of a wide range of values, tested at nominal rating show that the lower values more rigorously obey the cubic relationship, whereas higher values tend in some cases towards a power law between the square and the cubic forms. This suggests that in the latter cases, where high test voltages are applied, coefficients of a higher order than the third are playing a significant part in the current/voltage relationship.

Fig. 2. Commercially available test set for the measurement of third harmonic distortion

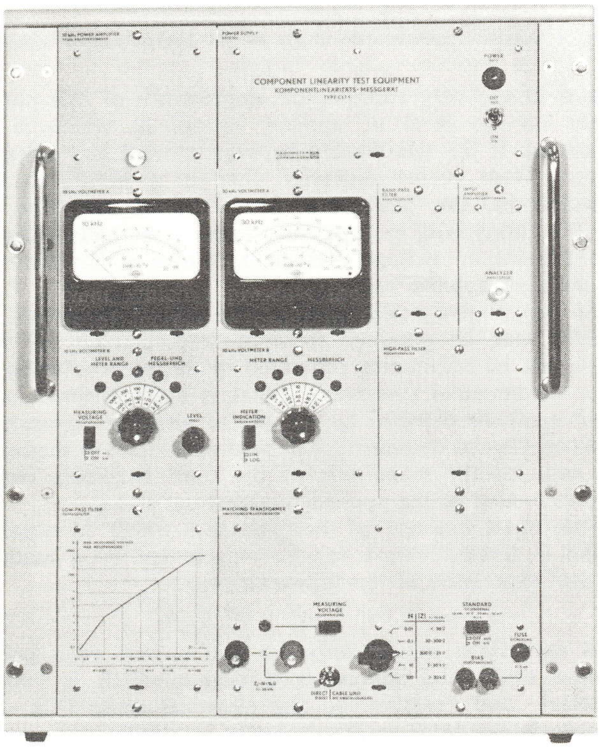


TABLE 1

Non-Linearity of Current/Voltage Characteristic

$$i=\alpha v + \beta v^3 + \gamma v^5 + \delta v^7 + \ldots$$

When $v=V\sin\omega t$

then $I=\alpha V\sin\omega t + \beta V^3\sin^3\omega t + \gamma V^5\sin^5\omega t + \delta V^7\sin^7\omega t + \ldots$

or in terms of fundamental 3rd harmonic 5th harmonic 7th harmonic

$$\begin{aligned} I= & \ \alpha V\sin\omega t \\ & +\beta V^3\left(\tfrac{3}{4}\sin\omega t - \tfrac{1}{4}\sin 3\omega t\right) \\ & +\gamma V^5\left(\tfrac{5}{8}\sin\omega t - \tfrac{5}{16}\sin 3\omega t + \tfrac{1}{16}\sin 5\omega t\right) \\ & +\delta V^7\left(\tfrac{35}{64}\sin\omega t - \tfrac{21}{64}\sin 3\omega t + \tfrac{7}{64}\sin 5\omega t - \tfrac{1}{64}\sin 7\omega t\right) \end{aligned}$$

Fourier Series for Symmetrical Non-Linearity up to the Seventh Order

$$\begin{aligned} I= & \left(\alpha V + \tfrac{3}{4}\beta V^3 + \tfrac{5}{8}\gamma V^5 + \tfrac{35}{64}\delta V^7\right)\sin\omega t. \\ & -\left(\tfrac{1}{4}\beta V^3 + \tfrac{5}{16}\gamma V^5 + \tfrac{21}{64}\delta V^7\right)\sin 3\omega t. \\ & +\left(\tfrac{1}{16}\gamma V^5 + \tfrac{7}{64}\delta V^7\right)\sin 5\omega t. \\ & -\left(\tfrac{1}{64}\delta V^7\right)\sin 7\omega t. \end{aligned}$$

A cubic expression is, however, a very useful approximation for all cases. A suitable unit for measuring the distortion of the device is given by expressing the ratio of the third harmonic, in microvolts, to the cube of the applied fundamental in volts (both in r.m.s.). In line with accepted practice for the expression of the current noise of a resistor, in terms[3] of a 'Noise Index' (n.i.) where:

$$n.i. = 20\log\frac{r.m.s.\ microvolts\ noise\ in\ a\ frequency\ decade}{d.c.\ volts\ applied}$$

a similar unit has been derived to permit more convenient analysis of the measurement of distortion.

Third Harmonic Index (t.h.i.) =

$$20\log\frac{r.m.s.\ microvolts\ of\ third\ harmonic}{(r.m.s.\ volts\ of\ applied\ fundamental)^3}$$

It is found that, in the range of fixed resistors of a variety of different types and values, this unit ranges from about −80 to −20dB.

The level of non-linearity varies considerably in a batch of nominally identical resistors. The use of a logarithmic unit is advantageous as the magnitude of the t.h.i. approximates to a normal distribution. This permits the use of statistical parameters and facilitates the application of control to this particular variable. Table 2 illustrates a series of grouped frequency tables for the t.h.i. of $\frac{1}{2}$W resistors of different types and values. Largely due to the use of the 'cube of the applied voltage' in the definition of t.h.i. it is found that this parameter *decreases* with increasing resistance value. This contrasts with the noise index which usually *increases* with resistance value. In any batch of resistors the magnitude of both t.h.i. and n.i. have remarkably similar distributions. Both are approximately normal when such logarithmic units are used, but fre-

TABLE 2

DISTRIBUTION OF T.H.I. FOR ½ WATT FILM RESISTORS OF VARIOUS VALUES

VALUE	10Ω		100Ω		1kΩ		10kΩ		100kΩ		1MΩ	
RESISTOR TYPE	OXIDE	CARBON	OXIDE	CARBON	OXIDE	CARBON	OXIDE	CARBON	OXIDE	CARBON	OXIDE	CARBON
T.H.I. (dB)												
+10 to + 5·1	2		1									
+ 5 + 0·1	98	2										
0 − 4·9		85										
− 5 − 9·9		13										
−10 −14·9			2									
−15 −19·9			1									
−20 −24·9			12	4								
−25 −29·9			47	8								
−30 −34·9			22	48		4						
−35 −39·9			12	40	1	5	1	1				
−40 −44·9			3		6	11	2	18				
−45 −49·9					14	15	9	81	1			
−50 −54·9					19	41	11		2	3		
−55 −59·9					34	24	17		2	13		
−60 −64·9					21		56		4	23		6
−65 −69·9					5		4		10	36		63
−70 −74·9									24	22		31
−75 −79·9									48	2		
−80 −84·9									9			

quently indicate some skewness which incorporates a 'tail' extending into the higher levels.

The value of the t.h.i. of any resistor, unlike its 'voltage coefficient', is practically independent of the level of applied test voltage. Most investigations of the harmonic content of the distortion introduced by the non-linearity of a fixed resistor suggest that the second harmonic and odd harmonics higher than the third are small compared with the latter. Thus for most purposes including the routine checking of the performance of resistors, a measurement of the third harmonic ratio and its expression as a third harmonic index is quite adequate. Commercial apparatus has been developed for the measurement of distortion in terms of the magnitude of the third harmonic component (Fig. 2).

If a more detailed investigation of the behaviour of the resistor is required it may be found that this method of assessing non-linearity is not entirely free from interference due to the temperature sensitivity of the resistor element. The self heating of the element in association with the temperature coefficient can produce a symmetrical non-linearity between current and voltage which results in a polynomial expression utilizing only the odd powers as in equation (1). In this case, however, there would be some slight difference in phase between the harmonic voltages due to true ohmic non-linearity and those resulting from the heating effect. For a thorough investigation apparatus would be required which could measure the amplitude of higher order harmonics and give some indication of their relative phase.

It was noted above that only those types of fixed resistor which incorporate a semiconducting material as the negative element exhibit ohmic non-linearity or current noise. A simplified explanation can be given concerning the basic physical mechanisms which give rise to these two effects in the materials concerned. There is as yet no generally accepted complete explanation for the occurrence of excess or current noise but most theories have centred around the modulation in conductivity which arises when current carriers pass into and out of the conduction bands. A similar general picture cannot be utilized to explain non-linearity as there is no reason why any such interchange of carriers should be dependent upon the magnitude of the applied voltage. Those regions where the energy levels are disturbed, as may occur due to the existence of surface states or at 'junctions' within the material, may be regarded both as a potential source of current noise and of non-linearity or variation of conductivity with applied potential. In other words, while current noise could occur in a semiconducting material without there necessarily being any form of discontinuity or junction, non-linearity can only occur in regions where some form of potential barrier has been produced.

Although a junction between two regions of the material, where different energy levels or different levels of carrier concentration exist, may clearly involve ohmic non-

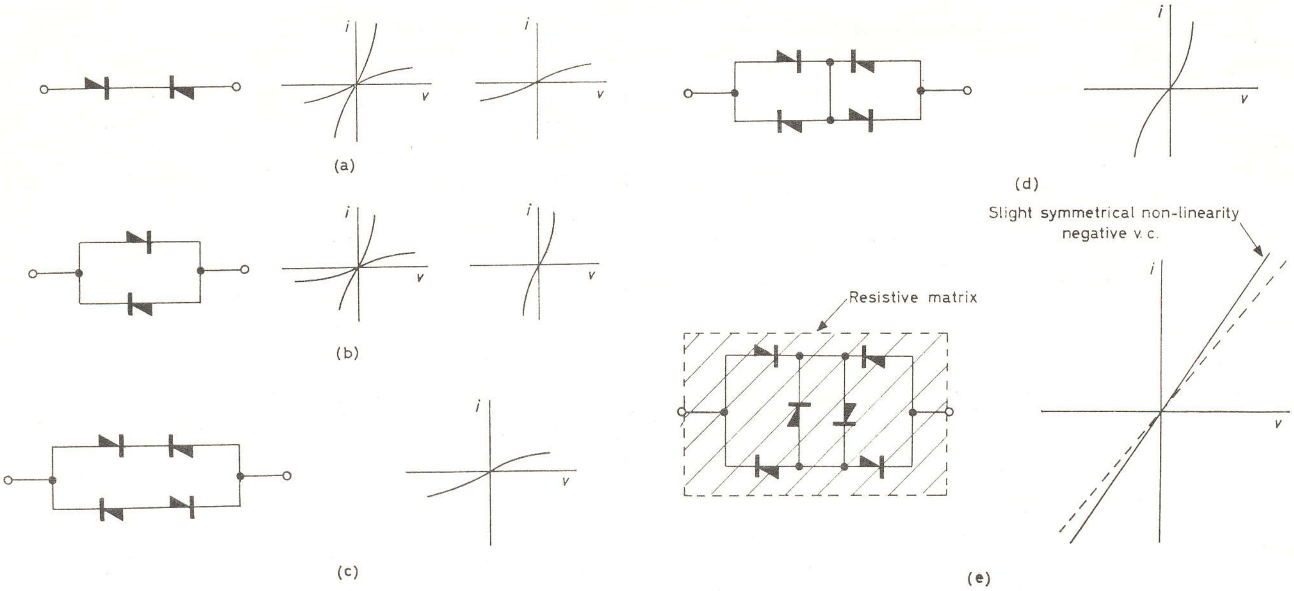


Fig. 3. Combination of rectifying junctions to synthesize the response of a fixed resistor

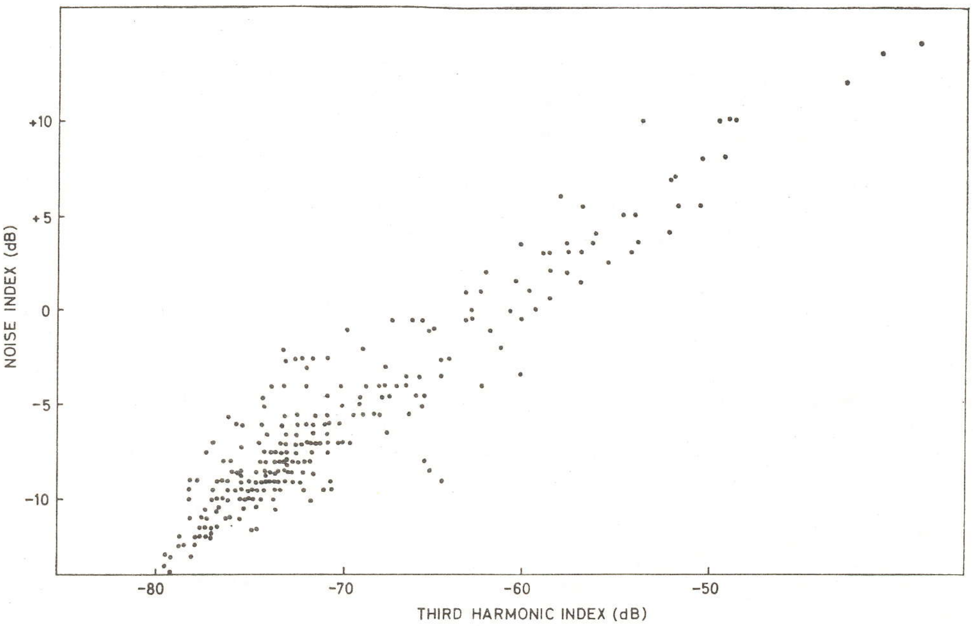
linearity, such a single junction will have an assymmetrical characteristic. This will occur whether the 'junction' occurs between p and n material or n and n+ material or between any regions where the energy levels or carrier concentrations differ. Fixed resistors, in contrast, exhibit a small degree of non-linearity of a symmetrical form. The resistive element consists of an amorphous or fine-grain polycrystalline aggregate of the semiconducting material concerned. In such a material the 'junctions' which may occur are likely to be widely distributed both in regard to the magnitude of the potential barriers and also with regard to the position and direction which they occupy in the electrical matrix. The resulting series-parallel combination of a wide variety of junctions can be shown to result in a slightly non-linear characteristic but one which, due to the even chance of a barrier appearing in one direction or in the other, has a reasonably symmetrical form. This simplified picture suggests that the basic source of ohmic non-linearity is the complex random set of quasi-junctions which are distributed throughout the resistive material.

The illustrations given in Fig. 3 show how a system of simple rectifying junctions can be built up to give an assembly whose response begins to approximate to the *i*/*v* characteristics of a resistive film. Figs. 3(a) and 3(b) represent two opposite junctions connected respectively in series and parallel. The former gives a high impedance unit with a positive voltage coefficient and the latter a low impedance unit with a negative coefficient. Similar differences in characteristics are found between Figs. 3(c) and 3(d) and here it can be seen that the provision of a cross link produces an all-important difference in the response of the assembly. Bearing in mind that the resistive film being simulated has a very low order of non-linearity but that it does invariably have a negative voltage coefficient, one must conclude that the 'cross-links' in the resistive film, which must certainly exist in practice, are of very great importance.

Fig. 3(e) illustrates a more complex assembly of rectifying junctions having the required negative coefficient and if this is now imagined to incorporate many additional resistor cross links, or preferably to be virtually immersed in a continuous resistive matrix, then the *i*/*v* characteristic is further modified and approaches the known behaviour of actual fixed resistors. A qualitative explanation of non-linearity of this nature can be utilized to explain the existence of ohmic non-linearity in both solid composition and film resistors.

The magnitude of the current noise of a film resistor

Fig. 4 The relationship between third harmonic distortion and current noise in a group of over 400 cracked-carbon resistors



and its dependence on the thickness, width and length of the resistor track has been studied previously[4]. It appears that the observed facts can be explained if it is assumed that current noise is produced at a large number of discrete sources distributed throughout the material of the film. The behaviour of the system can then be analysed in terms of a number of incoherent noise generators in a conducting matrix. The series and parallel combinations of random generators coupled with the series resistance and shunting effects of the matrix suffice to explain the observed effects.

The generation of noise throughout all parts of the material of the film due to the transition of carriers to and from the conduction band must still be regarded as a possible explanation of the phenomenon. Nevertheless, this picture of discrete random noise generators is very similar to the idea of a distribution of the 'junctions' which can be the source of non-linearity. The statistical relationship between current noise and non-linearity lends some emphasis to the idea that the major source of noise may correspond to the sole source of non-linearity at quasi-junctions dispersed throughout the film.

It would therefore be expected that there will be a correlation between the current noise and the ohmic non-linearity of a fixed resistor. A great deal of evidence has been compiled to illustrate this correlation and Fig. 4 shows the relationship between these two parameters in a group of over 400 ¼W 500kΩ carbon film resistors where the correlation coefficient was found to be 0·90.

In the case of a film resistor both phenomena can be affected by similar geometrical factors. Similarly, if an irregularity occurs at some point on the film then both the current noise and the non-linearity may be increased above the level associated with similar resistors without such a defect. The latter may be a possible subsequent cause of inferior performance and a preliminary measurement of noise or non-linearity may permit the elimination of this potential failure from the batch of resistors. Such a procedure must not be regarded as a complete guarantee against resistor failure as may sometimes have been suggested, but does go some way towards improving the reliability of the product. The choice as to whether current noise or non-linearity should be used for this purpose depends on particular circumstances. An important criterion is the reliability with which the instrument will operate on an automatic production line in carrying out measurements rapidly without interference from neighbouring equipment. It now appears that non-linearity test sets (Fig. 2) may well prove to be preferable to noise test sets in these respects.

Conclusion

The measurement of distortion given by the magnitude of the third harmonic component has been found to be a most convenient method of assessing the ohmic non-linearity of fixed resistors. Over a wide range of resistance values and at different levels of applied voltage it gives a repeatable measurement which is less affected by other resistor parameters than a measurement of so-called voltage coefficient. The latter parameter may well have lost some of its original popularity for reasons which have been explained. It is likely that a measurement of non-linearity by reference to the third harmonic distortion particularly when expressed in the form of a 'third harmonic index' will find greater future application. Such techniques have considerable value in laboratory investigations into resistor quality, and versatile apparatus of high sensitivity with the possibility of measuring the 3rd and 5th harmonic and their phase relative to the applied fundamental would be useful. In addition, however, there is the need for reliable lower-priced production line equipment to measure rapidly and reject automatically any resistor having a level of non-linearity greater than a prescribed limit. Finally, there is the likely future use of third harmonic distortion, perhaps in the form of the new parameter t.h.i. in resistor specifications. This could replace previous attempts to measure a voltage coefficient and would be complimentary to the growing use of a noise index with which the t.h.i. has such a significant correlation. An indication of the future areas of application of distortion measuring apparatus is given in Table 3.

TABLE 3

Third-Harmonic Distortion Measurements—Future Applications	
(i)	Laboratory investigations into resistor quality.
	Measurement of i/v symmetry, 3rd & 5th harmonics, and relative phase to distinguish t.c. etc.
	Versatile apparatus of high sensitivity.
(ii)	Component specifications,—replacing "voltage coefficient."
	Use of t.h.i. complimentary to noise index.
	Reliable and repeatable measurements under known conditions.
(iii)	Automatic production test,—rejection at given level.
	Data accumulation (on log. scale, i.e. t.h.i.) for process control.
	Robust lower-price apparatus—rapid measurement free from interference.

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