Some Properties of Negative Feedback Amplifiers*

By L. I. Farren, Wh.Sch., A.C.G.I., D.I.C.

(Communication from the Staff of the Research Laboratories of The General Electric Company, Limited, Wembley, England)

SUMMARY.—This paper deals with a subject which is of increasing interest to communication engineers. Various methods of applying negative feedback to thermionic amplifiers are discussed, together with the modifications to the amplifier characteristics produced thereby.

Unwanted phase shifts at very high and very low frequencies will often cause a feedback amplifier to oscillate, if the amount of feedback applied is large; methods of design are given, whereby the maximum possible amount of feedback may be applied without causing self oscillation.

The effect of negative feedback on harmonic distortion is discussed with reference to the case of an amplifier, the output valve of which has a quadratic relationship between grid voltage and anode current.

Contents

1. Introduction

ALTHOUGH the principle of negative feedback has been known for many years, it required the development of the modern high gain valves to make it useful. It then became apparent that the principle was of great importance, for the development among other things of Carrier Telephony, and in 1934 H. S. Black [1] wrote a paper which described this application. Prior to Black's publication, a paper was written by H. Nyquist [2] on the general theory of regeneration; at the time it appeared to be of mathematical interest only with very little practical application, but Peterson, Kreer and Ware [3] have since shown that amplifiers employing negative feedback obey exactly the laws derived by Nyquist.

Since these publications appeared, others have been written dealing with certain aspects of the subject, including in some cases design details of amplifiers for special purposes.†

However, many properties of negative feedback amplifiers which might be of use to prospective designers, have been dealt with in far too general a manner to be converted easily into practical figures.

It is the purpose of this article to describe certain properties of negative feedback amplifiers and show how these properties may be utilised.

2. Elementary Theory[1]

Fig. 1 shows a schematic diagram of an amplifier with negative feedback applied.

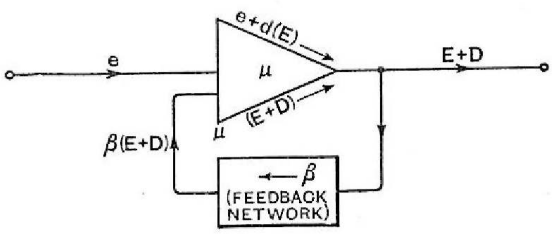


Fig. 1.—*Schematic diagram of amplifier and feedback network.*

The terms have the following significance:

e = Signal input voltage.

$$\mu = \frac{\text{Output voltage [excluding } d(E)]}{\text{Voltage on the first grid.}}$$

E = Signal output voltage.

$$\beta = \frac{\text{Voltage fed back to first grid.}}{\text{Output voltage.}}$$

* MS. accepted by the Editor, April, 1937.

† References 12, 13, 14 and 15 refer to publications which have appeared since MS. was accepted.

$d(E)$ = Distortion voltage generated in amplifier.

D = Distortion voltage in the output.

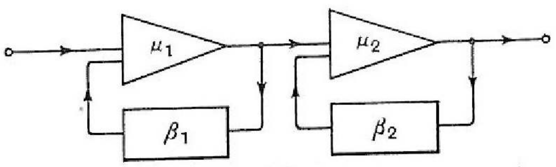
$$\therefore E + D = \mu e + d(E) + \mu\beta[E + D]$$

$$\text{or } E + D = \frac{\mu e}{1 - \mu\beta} + \frac{d(E)}{1 - \mu\beta} \quad \cdots \quad (1)$$

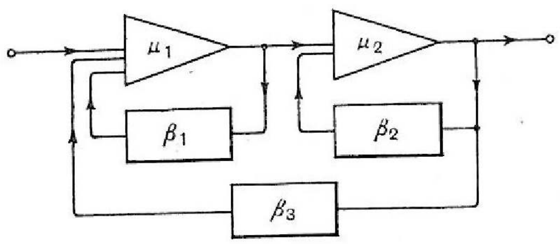
Hence by the application of negative feedback, the amplification has been reduced by a factor $(1 - \mu\beta)$ and the distortion $d(E)$ for a given output E has been reduced by the same factor.

Multiple Feedback.

In some cases it is considered preferable, in multi-stage amplifiers, to split the feedback; e.g. as shown in Fig. 2(a) and (b).



(a)



(b)

Fig. 2.—*Multiple feedback.*

In the case illustrated in Fig. 2(a), the overall amplification is given by

$$\mu = \frac{\mu_1\mu_2}{(1 - \mu_1\beta_1)(1 - \mu_2\beta_2)} \quad \cdots \quad (2)$$

In Fig. 2(b) the overall amplification is given by

$$\mu = \frac{\left(\frac{\mu_1}{1 - \mu_1\beta_1}\right)\left(\frac{\mu_2}{1 - \mu_2\beta_2}\right)}{1 - \beta_3\left(\frac{\mu_1}{1 - \mu_1\beta_1}\right)\left(\frac{\mu_2}{1 - \mu_2\beta_2}\right)} \quad \cdots \quad (3)$$

$$\text{or } \mu = \frac{\mu_1\mu_2}{(1 - \mu_1\beta_1)(1 - \mu_2\beta_2) - \mu_1\mu_2\beta_3} \quad \cdots \quad (4)$$

3. Methods of Applying Negative Feedback

It is of the utmost importance to consider the various methods of applying negative feedback, since a large number of methods are available, each of which endows the amplifier with some new property which may, or may not, be advantageous.

For the moment we will not consider how the feedback voltage is injected into the input circuit, but merely consider the method of tapping it from the output.

Three alternatives are available, which may be designated "current" feedback, "voltage" feedback and "bridge" feedback.

"Current" feedback implies that the feedback voltage is proportional to the output current; "voltage" feedback gives a feedback voltage proportional to the output voltage, and "bridge" feedback is a combination of the two.

"Current" Feedback.

Fig. 3 shows a schematic diagram of an amplifier employing "current" feedback.

The feedback voltage is derived across a resistance R_c in series with the load Z. μ represents the numerical value of the voltage amplification from the grid of the first valve to the anode of the output valve which has an internal resistance R_a. The negative sign before μ (Fig. 3) is obtained most readily by having an odd number of amplifying stages.

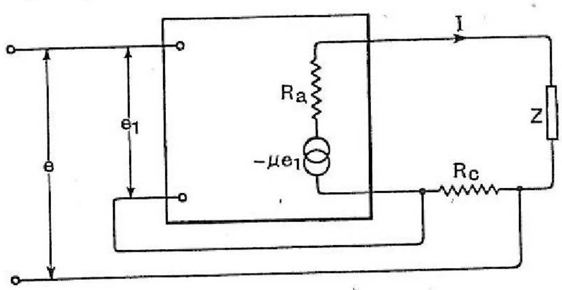


Fig. 3.—*"Current" feedback.*

We have

$$I = \frac{-\mu e_1}{R_a + Z + R_c} \quad \cdots \quad \cdots \quad (5)$$

But $e_1 = e + R_c I$

$$\therefore \quad I = \frac{-\mu e}{R_a + R_c(1 + \mu) + Z} \quad \cdots \quad (6)$$

If μ is made sufficiently large we have

$$I = -\frac{e}{R_c}.$$

Hence the output current, for a given input voltage is sensibly independent of R_a, Z and μ, and depends only on R_c.

At the same time the output impedance has been increased from R_a to $R_a + R_c(1 + \mu)$. By "output impedance" we mean the impedance which would be measured looking back into the output terminals, i.e. the internal impedance of the generator.

This property can therefore be made use of, in some cases, for matching a low impedance valve to a high impedance load, by making

$$Z = R_a + R_c(1 + \mu)$$

"Voltage" Feedback.

Fig. 4 shows a schematic diagram of an amplifier employing "voltage" feedback. The feedback voltage is derived from a tapping on a resistance R shunted across Z; R is considered $\gg Z$.

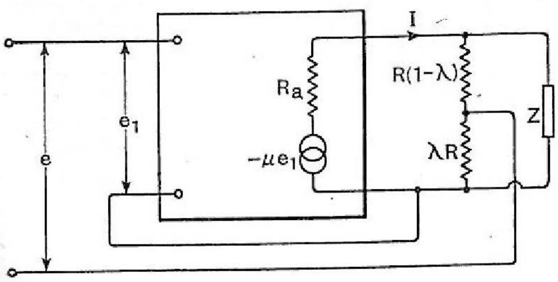


Fig. 4.—"*Voltage*" *feedback.*

We may write

$$I = \frac{-\mu e_1}{R_a + Z} \quad \cdots \quad \cdots \quad \cdots \quad (7)$$

But $\quad e_1 = e + \lambda ZI$

$$I = \frac{-\mu e}{R_a + Z(1 + \mu\lambda)} \quad \cdots \quad \cdots \quad \cdots \quad (8)$$

If $\mu\lambda$ is $\gg 1$

$$I = -\frac{e}{\lambda Z}$$

$$\text{Output voltage} = IZ = -\frac{e}{\lambda} \quad \cdots \quad (9)$$

Thus the output voltage for a given input voltage is sensibly independent of μ, R_a and Z and depends only on λ.

At the same time the output impedance has been reduced from R_a to $\frac{R_a}{1 + \mu\lambda}$; this property can therefore be employed for matching a high impedance output valve to a low or medium impedance load.

"Bridge" Feedback.

Fig. 5 shows a schematic diagram of an amplifier employing "bridge" feedback.

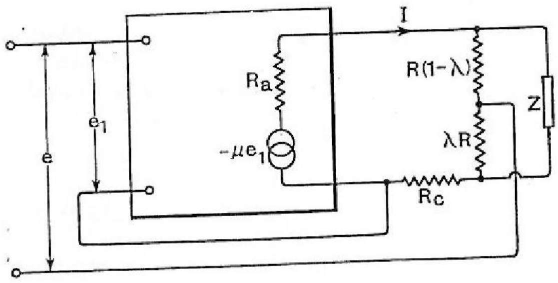


Fig. 5.—"*Bridge*" *feedback.*

The feedback voltage is the vectorial sum of the voltages across R_c and λR; the voltage across R_c is proportional to the current in the load Z, and the voltage across λR is proportional to the voltage across Z.

Assuming once more that $R \gg Z$, we have

$$e_1 = e + R_c I + \lambda ZI$$

$$\therefore I = \frac{-\mu[e + R_c I + \lambda ZI]}{R_a + R_c + Z}$$

$$\text{or } I = \frac{-\mu e}{R_a + R_c(1 + \mu) + Z(1 + \mu\lambda)} \quad (10)$$

Approximating as before,

$$I = \frac{-e}{R_c + \lambda Z} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (11)$$

In this case the output current and voltage are independent of μ and R_a, but dependent on Z. The output impedance, from equation (11) is equal to $\frac{R_c}{\lambda}$.

Hence the use of bridge feedback renders the output impedance constant, if R_c and λ are constant, but both the output current and voltage vary with frequency, if Z is frequency dependent.

The extent to which the bridge method of feedback stabilises the output impedance, and one important justification for its use is indicated in the example given below.

Experimental results are also given. Fig. 6 shows the equivalent circuit of a triode valve, with an amplification factor μ and internal resistance R_a.

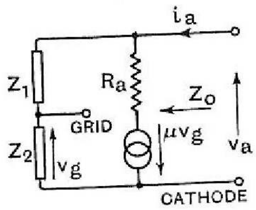


Fig. 6. — *Thermionic valve with anode-grid and grid-cathode impedance.*

Z_1 represents the anode-grid impedance (normally a pure capacity) and Z_2 represents the grid cathode impedance, which includes impedances associated with the preceding amplifier stage.

Assuming that $Z_1 + Z_2 \gg R_a$; if a voltage v_a is applied to the output terminals as shown, a voltage v_g will appear between the grid and cathode through the potentiometer comprising Z_1 and Z_2.

$$\therefore i_a = \frac{v_a + \mu v_g}{R_a}$$

$$= \frac{v_a + \mu \left[\frac{Z_2}{Z_1 + Z_2}\right] v_a}{R_a}$$

$$\therefore Z_0 = \frac{v_a}{i_a} = \frac{R_a(Z_1 + Z_2)}{Z_1 + Z_2(1 + \mu)} \quad \ldots \quad (12)$$

Z_1 and Z_2 are generally reactive, and only if their phase angles are identical at all frequencies is Z_0 constant. As this condition rarely arises, in general Z_0 will vary with frequency.

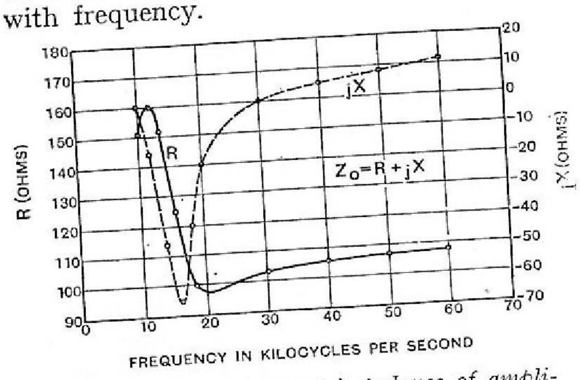


Fig. 7.—*Measured output impedance of amplifier without negative feedback.*

Fig. 7 shows the results of measuring Z_0 on an amplifier with a low impedance triode output valve. The valve fed a transformer of step down ratio 2.45 : 1, and the impedance measurements were taken looking back into the secondary winding.

We see from Fig. 7 that both the resistive and reactive component of Z_0 vary greatly with frequency, the resonance effect at 16 kc/s being due to a coupling choke in the penultimate stage.

On applying "bridge" feedback, with the following values, $R_c = 51\ \Omega$, $\lambda = 1/20$, the results obtained for Z_0 were as shown in Fig. 8.

The resistive component of Z_0 is practically constant and the small reactive component comes from the inherent reactances present in the transformer, which are not eliminated by negative feedback.

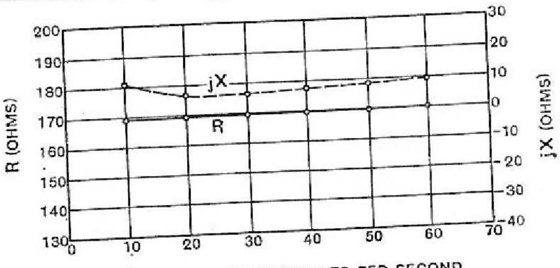


Fig. 8.—*Measured output impedance of amplifier with negative feedback applied.*

The "bridge" method of feedback is the most flexible form, since by making $R_c = 0$ or $\lambda = 0$, we obtain the "voltage" or "current" feedback circuits respectively.

Let us consider the general case when Z is dependent on frequency.

Case 1.—Use of "current" feedback to give constant output voltage.

If, instead of R_c (Fig. 3) we employ an impedance Z_c where $Z_c = KZ$ (K being constant), we have

$$I = -\frac{e}{KZ}$$

$$\text{and } E = IZ = -\frac{e}{K} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

We may thus obtain constant output voltage with a high output impedance.

Case 2.—Use of "voltage" feedback to give constant output current.

Instead of $R(1 - \lambda)$ and λR in Fig. 4, replace these quantities by KZ and R_1 where $K \gg 1$.

$$\therefore \lambda = \frac{R_1}{R_1 + KZ} \quad .. \quad .. \quad .. \quad (14)$$

In general μ is very large and λ fairly small so that $\mu\lambda$ is greater than unity.

Hence we may approximate and write equation (14) as,

$$\lambda = \frac{R_1}{KZ} = \frac{K_1}{Z}$$

We thus have

$$I = \frac{-e}{\lambda Z} = \frac{-e}{K_1} \quad .. \quad .. \quad (15)$$

We may thus obtain constant output current with low output impedance.

Case 3.—Use of bridge feedback to give constant output voltage with constant output impedance.

Fig. 9 shows a form of circuit which will give the desired result.

The two connections which normally connect the output bridge to the input circuit are joined to the ends of impedances $K\lambda Z$ and KR_c in series and the feedback voltage is developed across $K\lambda Z$. We assume

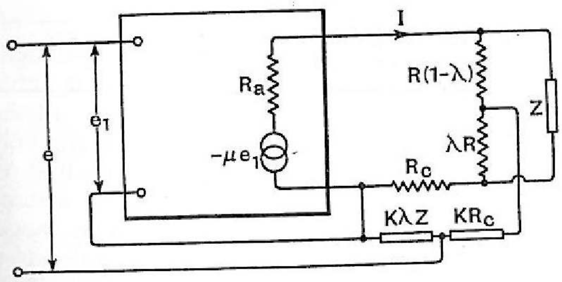


Fig. 9.—*Amplifier with constant output voltage and constant output impedance.*

that $K\lambda Z + KR_c$ is $\gg \lambda R + R_c$, so that the added potentiometer serves as a voltage divider taking no appreciable current.

Rewriting equation (11) we have

$$I = \frac{-e}{\dfrac{K\lambda Z}{K\lambda Z + KR_c}[R_c + \lambda Z]} \quad .. \quad (16)$$

$$E = IZ = \frac{-e}{\lambda}$$

The output impedance $= \dfrac{R_c}{\lambda}$ is unaltered, but the output voltage has been rendered independent of Z.

Case 4.—Use of bridge feedback to give constant output current with constant output impedance.

This result may be achieved if a circuit is used similar to Fig. 9, except that the arms of the added potentiometer are interchanged.

Rewriting equation (11) we have

$$I = \frac{-e}{\dfrac{KR_c}{K(R_c + \lambda Z)}[R_c + \lambda Z]} = \frac{-e}{R_c} \quad .. \quad (17)$$

The output impedance $= \dfrac{R_c}{\lambda}$ is unaltered, but the output current has been rendered independent of Z.

Case 5.—Use of bridge feedback to give constant voltage output with output impedance = load impedance (Z).

A circuit which will give the desired result is similar to Fig. 5 except that R_c is replaced by an impedance λZ.

Rewriting equation (11) we have

$$I = \frac{-e}{\lambda Z + \lambda Z} = \frac{-e}{2\lambda Z} \quad .. \quad .. \quad (18)$$

$$E = IZ = -\frac{e}{2\lambda}$$

The output impedance $= \dfrac{\lambda Z}{\lambda} = Z$, and the output voltage is constant.

This circuit is of use in cases where it is desired to match the output impedance of the amplifier to a load impedance which varies with frequency; this is necessary in many cases where reflections, occurring as the result of an impedance mismatch, must be avoided.

Case 6.—Use of bridge feedback to give constant current output with output impedance = load impedance (Z).

Fig. 10 shows a circuit for achieving this result. The new value of λ is $\dfrac{KR_c}{KZ} = \dfrac{R_c}{Z}$, K is made $\gg 1$.

Rewriting equation (11) we get

$$I = \frac{-e}{R_c + \dfrac{R_c}{Z}\cdot Z} = \frac{-e}{2R_c} \quad .. \quad .. \quad (19)$$

The output impedance $= \dfrac{R_c}{\lambda} = Z$, and the output current is constant.

Fig. 11 shows an alternative circuit for producing the same result.

$$I = \frac{-e}{\frac{KR_1}{KZ}[\lambda Z + \lambda Z]} = \frac{-e}{2\lambda R_1} \quad \cdots \quad (20)$$

Output impedance $= \frac{\lambda Z}{\lambda} = Z.$

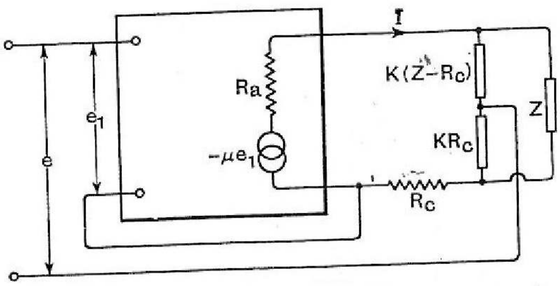


Fig. 10.—*Amplifier with constant output current and output impedance = Z (load impedance).*

4. Phase Shift and Instability

We have seen, from equation (1), that if an amplifier with an amplification factor μ has a fraction of its output fed back to the input circuit, the new amplification factor is given by

$$\mu_F = \frac{\mu}{1 - \mu\beta}$$

If the amplifier comprises a number of stages, each containing reactive components, μ will be some function of frequency. Ignoring the case of D.C. amplifiers, in general for $\omega = 0$ and $\omega = \infty$, $\mu = 0$.

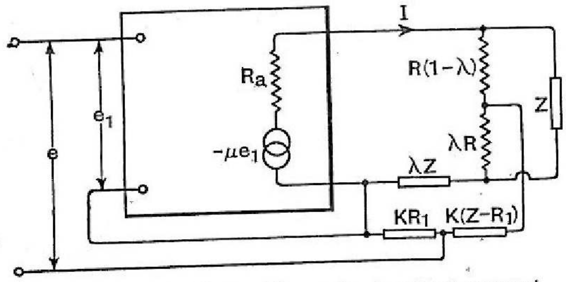


Fig. 11.—*Amplifier with constant output current and output impedance = Z (load impedance).*

Over the desired range of frequencies $\mu\beta$ is made negative, generally by making μ negative. This may be accomplished readily by using an odd number of valve stages, although methods are known whereby an even number of stages may be used.

Since, however, μ is dependent on frequency, it is quite possible for it to be positive and real at certain frequencies. The value of μ under such conditions determines the maximum value of β which can be tolerated. If μ_K is the critical value of μ, for which μ is positive, then if $\mu_K\beta \geqslant 1$ the amplifier is unstable. Actually, Nyquist[2] has shown that this assumption is not rigorously true, but for most cases it is correct.

Thus, if we denote by β_K the maximum value that we can assign to β, we have

$$\mu_K\beta_K = 1.$$

We see therefore that the minimum value obtainable for μ_K determines the maximum value allowable for β_K and hence determines the amount of feedback which may safely be applied.

The problem of designing a circuit to give the smallest unavoidable value of μ_K is far too complicated to be dealt with generally. However, two methods which may be applied in a number of cases will now be given in detail.

5. Method of Proportioning the Stages

This analysis is due to Dr. Werrmann[4].

Let us consider the case of a multi-stage resistance-capacitance coupled amplifier. Each stage of the amplifier would be of the form shown in Fig. 12(a).[11] The symbols have the following meaning.

e = Grid voltage applied to the stage.
g = Mutual conductance.
C_{ac} = Anode cathode capacitance.
R_a = Internal resistance of valve.
R_L = Load resistance.
C_1 = Coupling condenser capacitance.
R_g = Grid cathode resistance of next valve.
C_{gc} = Grid cathode capacitance of next valve.
E = Voltage applied to grid of the next valve.

The capacitance C_{ac} is usually very small ($< 10\mu\mu F$) and has an effect only at high frequencies; hence we can consider it as being effectively in parallel with C_{gc} giving a resultant shunt capacitance

$$C_g = C_{ac} + C_{gc}$$

In general we are dealing with screen grid or screened pentode valves in which the anode grid capacitance is very small; for this reason it has been ignored.

We may write

$$\frac{E}{e} = -g\frac{j\omega C_1 R_a R_L R_g}{(R_a + R_L - \omega^2 R_a R_L R_g C_1 C_g) + j[\omega R_g(R_a + R_L)(C_1 + C_g) + \omega R_a R_L C_1]} \quad .. \quad .. \quad (21)$$

The expression by which g is multiplied has the form of a complex impedance, and it can be shown that this impedance may be represented in the circuit of Fig. 12(b); R, L and C in parallel form the desired

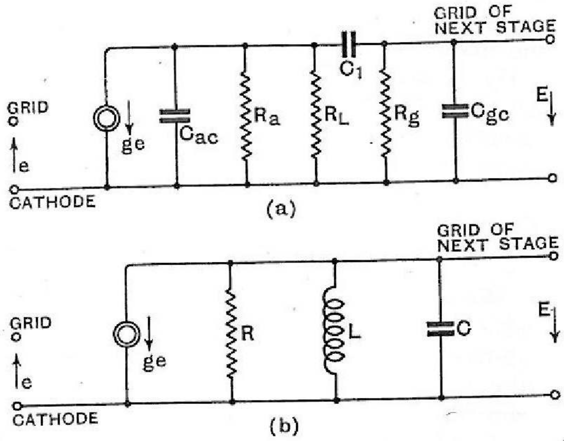


Fig. 12 (a).—*Equivalent circuit of thermionic valve and resistance-capacitance coupling.* (b).—*Circuit which is equivalent electrically to* (a).

impedance. The first conception of a resistance capacitance coupled amplifier as a tuned amplifier was due to Luck.[5] The following relations are true.

$$\left.\begin{aligned} \frac{1}{R} &= \frac{1}{R_a} + \frac{1}{R_L} + \frac{1}{R_g} + \left(\frac{1}{R_a} + \frac{1}{R_L}\right)\left(\frac{C_g}{C_1}\right) \\ L &= C_1 R_g\left[\frac{R_a R_L}{R_a + R_L}\right] \\ C &= C_g \end{aligned}\right\} \quad .. \quad (22)$$

We may now study the effect of proportioning the relative magnitudes of R, L and C.

For a single stage, as shown in Fig. 12(b), the amplifying factor of the stage is given by $\mu = \dfrac{E}{e}$

$$\therefore \mu = \frac{E}{e} = \frac{-g}{\dfrac{1}{R} + \dfrac{1}{j\omega L} + j\omega C} \quad .. \quad (23)$$

Consider the case of an amplifier with "r" stages (not necessarily identical).

We have $\mu = \mu_1 \cdot \mu_2 \cdot \mu_3 \cdot \cdot \cdot \cdot \mu_r$.

From equation (23) we may write

$$\mu = \prod_{n=1}^{n=r} \frac{-g_n}{\dfrac{1}{R_n} + \dfrac{1}{j\omega L_n} + j\omega C_n} \quad .. \quad (24)$$

By means of equation (24) it is possible to find one or more values of ω at which μ is real and positive; these values of μ have already been called μ_K (critical values).

Any particular case is most easily solved by graphical methods, as analysis would become tedious. Certain special cases may be solved however.

Case 1.—*r identical stages.*

From equation (24)

$$\mu = \left\{\frac{-g}{\dfrac{1}{R} + \dfrac{1}{j\omega L}\left(1 - \dfrac{\omega^2}{\omega_0^2}\right)}\right\}^r \quad .. \quad (25)$$

where $\omega_0^2 LC = 1$

When $\omega = \omega_0$, $\mu = \mu_0 = (-gR)^r$

μ_0 represents the maximum amplification obtainable, and since we normally employ an odd number of stages, μ_0 is negative.

From (25)

$$\mu = \mu_0\left[\frac{1}{\left\{1 + \dfrac{R\left(1 - \dfrac{\omega^2}{\omega_0^2}\right)}{j\omega L}\right\}^r}\right] \quad .. \quad (26)$$

The phase angle of μ may be determined from the phase angle of the denominator of the bracketed expression.

$$\text{Let} \quad \frac{R\left(1 - \dfrac{\omega^2}{\omega_0^2}\right)}{\omega L} = \tan\phi$$

$$\therefore \quad \mu = \mu_0\frac{\cos^r\phi}{[\cos\phi - j\sin\phi]^r}$$

$$= \frac{\mu_0\cos^r\phi}{\cos r\phi - j\sin r\phi} \quad .. \quad .. \quad (27)$$

With the coupling circuit as shown in Fig. 12(b) ϕ must lie between the limits $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

Hence $\cos^r \phi$ is always positive.

Hence from equation (27) assuming μ_0 negative, μ is real and positive when

$$\cos r\phi - j \sin r\phi = -1 \text{ or } \phi = \frac{\pi}{r}$$

$$\text{Thus } \mu_K = -\mu_0 \cos^r\left(\frac{\pi}{r}\right) \quad \dots \quad \dots \quad (28)$$

If β_K is the maximum fraction of the output which may be fed back, we may write

$$\beta_K = \frac{1}{\mu_K}$$

Hence the maximum gain reduction, or linearisation, which we can obtain is given by

$$1 - \mu_0\beta_K = 1 + \frac{1}{\cos^r\left(\frac{\pi}{r}\right)} \quad \dots \quad (29)$$

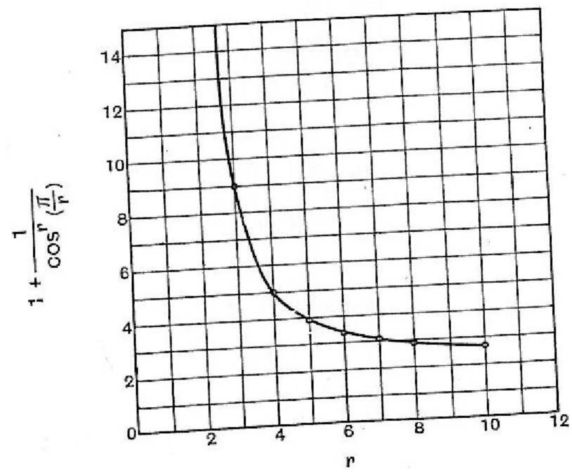


Fig. 13.—*Curve showing the dependence of the maximum possible amount of feedback on the number of stages (r), assuming identical stages.*

We see that the amount of linearisation possible is dependent on r; a curve relating r and $1 + \frac{1}{\cos^r\left(\frac{\pi}{r}\right)}$ is shown in Fig. 13.

With $r = 1$ (case of single stage amplifier), the critical frequency never occurs; with $r = 2$ it only occurs at $\omega = 0$ and $\omega = \infty$, in which case $\mu_K = 0$.

μ_K only becomes real for $r > 2$.

In other words, single stage and two stage amplifiers using a coupling circuit as shown, should never be unstable however much feedback is applied. In practice, of course, such factors as decoupling circuits and unwanted reactive components in the feedback path all cause undesirable phase shifts which may cause instability even in the case of a two-stage amplifier. From Fig. 13, we see that with 3 identical stages ($r = 3$), the maximum degree of linearisation is 9.

By making the 3 stages dissimilar in a definite manner, the maximum degree of linearisation obtained may be increased quite considerably.

Case 2.—The effect of dimensioning on the 3-stage case.

We have seen that the maximum value which $\mu_0\beta_K$ can have in the case of three identical stages is 8. We will now consider the case of three stages dimensioned differently.

Our first assumption is to give each stage the same resonant frequency $\omega_0 = \frac{1}{\sqrt{L_n C_n}}$, but at the same time we allow the quantities $\frac{L_n}{R_n}$ to be different, if necessary, for each stage.

Let us write

$$\omega_1 = \frac{R_1}{L_1}, \quad \omega_2 = \frac{R_2}{L_2}, \quad \omega_3 = \frac{R_3}{L_3}$$

$$\therefore \mu = \prod_{n=1}^{n=3} -g_n R_n \left\{ \frac{1}{1 + \frac{R_n}{j\omega L_n}\left(1 - \frac{\omega^2}{\omega_0^2}\right)} \right\} \quad (30)$$

$$= \mu_0 \prod_{n=1}^{n=3} \frac{1}{1 + \frac{\omega_n}{j\omega}\left(1 - \frac{\omega^2}{\omega_0^2}\right)} \quad \dots \quad (31)$$

where $\mu_0 = -(g_1 g_2 g_3)(R_1 R_2 R_3)$

The critical pulsatance ω_K is determined from the value of ω which makes μ real and positive (μ_K).

This is obtained from a solution of the equation

$$\prod_{n=1}^{n=3} \left\{ 1 - \frac{j\omega_n}{\omega}\left(1 - \frac{\omega^2}{\omega_0^2}\right) \right\} = A + j \cdot 0$$

For the imaginary component of the LH expression to be zero,

$$\frac{(\omega_1+\omega_2+\omega_3)\left(1-\frac{\omega^2}{\omega_0^2}\right)}{\omega} = \frac{\omega_1\omega_2\omega_3\left(1-\frac{\omega^2}{\omega_0^2}\right)^3}{\omega^3} \quad \ldots \quad \ldots \quad (32)$$

$$\text{or } \sqrt{\frac{\omega_1+\omega_2+\omega_3}{\omega_1\omega_2\omega_3}} = \frac{\left(1-\frac{\omega^2}{\omega_0^2}\right)}{\omega} \quad \ldots \quad (33)$$

The two values of ω obtained from equation (33) give the upper and lower values of pulsatance at which μ is real and positive. By making ω_0 the same for each stage, for each of these values of ω, μ has the same value.

Substituting (33) in (31) we get

$$\mu_K = \frac{\mu_0}{1-(\omega_1+\omega_2+\omega_3)\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}+\frac{1}{\omega_3}\right)} \quad \ldots \quad (34)$$

Let us assume, arbitrarily, that two of these pulsatances are fixed, say, ω_1 and ω_3.

Let us suppose that ω_2 may be varied between the limits ω_1 and ω_3; we have to find the values of ω_2 that make the expression $(\omega_1+\omega_2+\omega_3)\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}+\frac{1}{\omega_3}\right)$ a maximum.

As an example, suppose $\omega_3 = 5\omega_1$; Fig. 14 shows the variation of the magnitude of the above expression for values of ω_2 between ω_1 and $5\omega_1$.

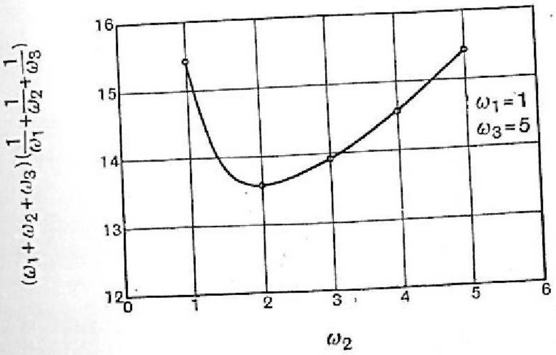


Fig. 14.—*Curve showing the effect of proportioning the stages in a three-stage amplifier.*

We see from Fig. 14 that the maximum linearisation (i.e. minimum value of μ_K) is obtained under the conditions $\omega_2 = \omega_1$ or $\omega_2 = \omega_3$.

Having now decided that ω_2 must equal either ω_1 or ω_3 we may study the effect of varying the ratio $\frac{\omega_3}{\omega_1}$.

Let $\omega_1 = a$, $\omega_2 = a$, $\omega_3 = ma$ where m is any number.

Substituting these values in (34) we get

$$\mu_K = \frac{\mu_0}{1-\frac{(1+2m)(2+m)}{m}} \quad \ldots \quad (35)$$

The maximum degree of linearisation

$$= 1 - \mu_0\beta_K$$
$$= 1 - \frac{\mu_0}{\mu_K}$$
$$= \frac{(1+2m)(2+m)}{m} \quad \ldots \quad \ldots \quad (36)$$

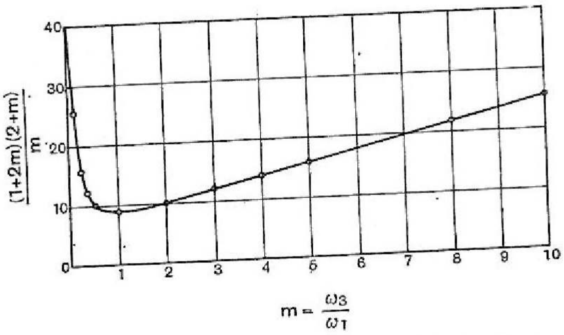


Fig. 15.—*Curve showing the effect of proportioning the stages in a three-stage amplifier.*

It is quite obvious that the same result would be achieved if $\omega_1 = a$, $\omega_2 = ma$, $\omega_3 = ma$.

Fig. 15 shows the variation of

$$\frac{(1+2m)(2+m)}{m}$$

plotted against m; we see that for $m = 1$ (i.e. three identical stages) the value is 9 (compare Fig. 13 for $r = 3$).

Summary of analysis.

From the foregoing we see that in order to be able to apply the maximum amount of negative feedback, the following rules must be adopted.

1. The resonant frequency ω_0 (as defined) must be the same for each stage.

2. If we characterise each stage by a lower pulsatance $\omega_a = \frac{R}{L}$ and an upper pulsatance

$$\omega_b = \frac{1}{CR}, \quad \omega_0 = \sqrt{\omega_a \omega_b}.$$

In the case of three stages, two of the lower pulsatances ω_a and two of the upper pulsatances ω_b must be equal, whilst the remaining lower and upper pulsatance respectively must be as different from these as possible.

6. Method of Subsidiary Feedback

This method is an alternative to the above for the purpose of allowing the maximum amount of feedback to be applied to an amplifier consistent with stability (6).

In a multi-stage amplifier, at least one of the stages is provided with a subsidiary feedback circuit, adapted to feed back a voltage which, over the working frequency range, is so small that the forward amplification is substantially unaffected. At frequencies outside this range, however, this voltage is increased so as to reduce considerably the gain of the stage and of the amplifier without introducing a phase shift liable to cause instability.

The method is termed subsidiary feedback because the feedback is common to a single stage as distinct from the main feedback circuit which connects the input and output of the whole amplifier.

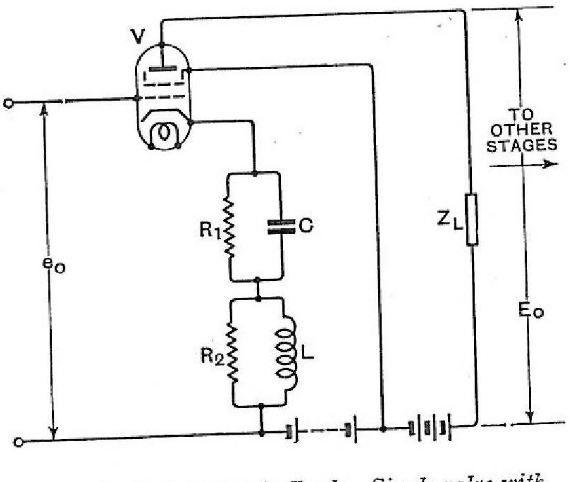


Fig. 16.—*Subsidiary feedback. Single valve with "current" feedback.*

Fig. 16 shows one circuit for achieving the desired result.

The valve V may be assumed to be a screen grid or pentode valve with a load impedance Z_L.

Subsidiary feedback is developed across an impedance in the cathode circuit comprising resistances R_1 and R_2, an inductance L and capacitance C as shown. If the valve constants are μ, R_a and $g = \frac{\mu}{R_a}$, and if the feedback impedance be called Z, we have

$$\left(\frac{E_0}{e_0}\right)_z = \frac{-\mu Z_L}{R_a + Z_L + Z(1 + \mu)} \quad .. \quad (37)$$

If R_a is $\gg Z_L$ and $\mu \gg 1$, then

$$\left(\frac{E_0}{e_0}\right)_z = \frac{-Z_L}{\frac{1}{g} + Z} \quad .. \quad .. \quad (38)$$

If there were no subsidiary reaction, Z would be zero and

$$\left(\frac{E_0}{e_0}\right)_{z=0} = -gZ_L$$

$$\therefore \quad \frac{\left(\frac{E_0}{e_0}\right)_z}{\left(\frac{E_0}{e_a}\right)_{z=0}} = \frac{1}{1 + gZ} = P + jQ \quad .. \quad (39)$$

If μ_z is the modulus of $P + jQ$ and θ the change in phase difference between E_0 and e_0 due to the introduction of Z, then

$$\left.\begin{aligned} \mu_z &= (P^2 + Q^2)^{\frac{1}{2}} \\ \tan\theta &= \frac{Q}{P} \end{aligned}\right\} \quad .. \quad .. \quad (40)$$

Let C shunted by R_1 be an impedance $A_1 + jB_1$ and L shunted by R_2 be an impedance $A_2 + jB_2$.

$$\therefore Z = A_1 + A_2 + j(B_1 + B_2)$$

$$P = \frac{1 + g(A_1 + A_2)}{[1 + g(A_1 + A_2)]^2 + g^2(B_1 + B_2)^2}$$

$$Q = \frac{-g(B_1 + B_2)}{[1 + g(A_1 + A_2)]^2 + g^2(B_1 + B_2)^2}$$

$$\left.\begin{aligned} \therefore \mu_z &= \left[\frac{1}{[1 + g(A_1 + A_2)]^2 + g^2(B_1 + B_2)^2}\right]^{\frac{1}{2}} \\ \tan\theta &= \frac{-g(B_1 + B_2)}{1 + g(A_1 + A_2)} \end{aligned}\right\} \quad .. \quad .. \quad (41)$$

By way of illustration, let us take the following values as an example.

$g = 0.008$ amps/volt
$R_1 = 100\ \Omega$
$R_2 = 1000\ \Omega$
$C = 1\mu F$
$L = 0.5$ mH

The following table shows the values of μ_z and θ for values of frequency from 10 c/s to 2 Mc/s.

Frequency c/s.	$A_1 + A_2$	$B_1 + B_2$	Change in Gain. μ_z in dB's.	Phase Shift θ
10	100	0	− 5.1	0
30	100	− 1.92	− 5.1	0
100	100	− 6.27	− 5.1	0
300	96.5	−18.6	− 4.9	4° 48′
700	83.9	−34.7	− 4.5	9° 24′
1,000	71.8	−45.6	− 4.2	13° 0′
1,500	53	−45.3	− 3.3	14° 12′
2,000	38.6	−42.3	− 2.5	14° 30′
3,000	21.9	−31.9	− 1.5	12° 18′
10,000	3.5	15.7	− .2	− 7° 0′
30,000	9.1	88.7	− 1.6	−33° 30′
60,000	34.4	180	− 5.5	−48° 6′
100,000	86.0	284	− 9	−53° 18′
200,000	310	450	−14	−45° 54′
300,000	420	498	−15.8	−39° 54′
500,000	710	452	−17.7	−28° 30′
1,000,000	890	290	−18.6	−16° 0′
2,000,000	975	155.2	−19.1	− 8° 0′

The working range of frequencies would extend from about 3 kc/s to 40 kc/s; over this range the change in gain due to subsidiary feedback is small.

Above this range the decrease in gain is fairly considerable and is not accompanied by a correspondingly large phase shift; in fact, the phase shift actually decreases at high frequencies and becomes zero at a frequency of infinity. In the example chosen, due to the particular values assigned to R_1 and C, the gain reduction at low frequencies is small, but by making $R_1 = R_2$ the gain reduction at very low frequencies will be the same as at very high frequencies.

This method does not modify the fundamental phase shift between E_0 and e_0 due to the phase angle of Z_L, but the main point in its favour is that a gain reduction is obtained with no appreciable increase in the phase shift which would be present if no feedback were applied.

An alternative method is shown in Fig. 17; the circuit is similar in some ways to Fig. 16, the difference lying in the fact that the subsidiary feedback is now of the "voltage" type.

Feedback voltage is applied across the bottom half of a potentiometer shunted across Z_L, consisting of resistances R_1, R_2 and R_3 capacitance C and inductance L,

Assume that Z has the same meaning as before and assume $R_3 + Z \gg Z_L$

$$\therefore \left(\frac{E_0}{e_0}\right)_z = \frac{-gZ_L}{1 + \frac{gZZ_L}{R_3 + Z}} \quad \ldots \quad \ldots \quad (42)$$

and
$$\frac{\left(\frac{E_0}{e_0}\right)_z}{\left(\frac{E_0}{e_0}\right)_{z=0}} = \frac{1}{1 + \frac{gZZ_L}{R_3 + Z}} \quad \ldots \quad \ldots \quad (43)$$

From this expression the gain reduction can be calculated if Z_L is known.

At very high frequencies, Z approximates to R_2 and equation (42) becomes

$$\left(\frac{E_0}{e_0}\right)_z = \frac{-gZ_L}{1 + KZ_L} \text{ where } K = \frac{-gR_2}{R_2 + R_3} =$$

constant.

It is possible in practice to make KZ_L large compared with unity, in which case

$$\left(\frac{E_0}{e_0}\right) = \frac{-gZ_L}{KZ_L} = \frac{-g}{K} \quad \ldots \quad \ldots \quad (44)$$

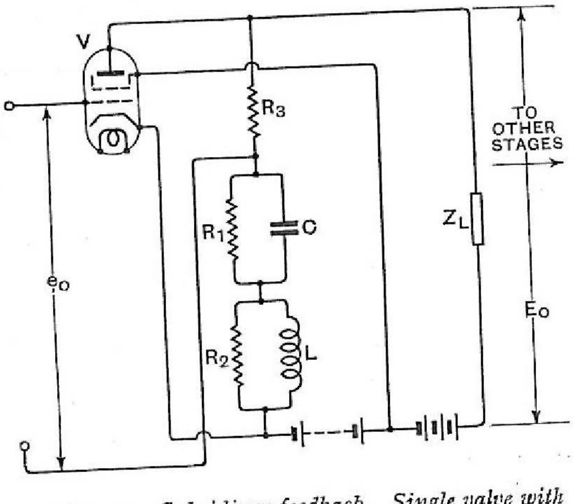


Fig. 17.—*Subsidiary feedback. Single valve with "voltage" feedback.*

Thus, at these high frequencies the voltage E_0 applied to the next stage is approximately 180 deg. out of phase with e_0, irrespective of the phase angle of Z_L; hence it approaches the ideal conditions. Similar reasoning holds for very low frequencies. Thus the arrangement shown in Fig. 17 has the advantage that at very high and very low frequencies it reduces the phase shift which would be characteristic of the stage if there were no subsidiary reaction, as well as giving a decrease in gain. Other methods which deal with the subject of improving the stability of feedback amplifiers by control of phase shift are given in references (7), (8) and (9).

7. Non-Linear Distortion in Negative Feedback Amplifiers

In the section on elementary theory we saw that if an amplifier, giving an output distortion voltage $d(E)$ for a given output E, has negative feedback applied to it, then for the same output E the distortion is reduced to $\frac{d(E)}{1 - \mu\beta}$.

This result can only be achieved by neglecting "secondary" distortion; in other words, the output distortion voltage which is fed back to the input and amplified is distorted once again and not amplified linearly as Black assumes. Admittedly this effect may be ignored for small outputs, but when the output valve is approaching the overload point e.g. when it is generating over 5 per cent. harmonic, the reduction in harmonic by the application of feedback may be smaller than one would expect theoretically. The rigorous treatment of the general case of distortion in a feedback amplifier has been given by R. Feldtkeller (10).

In the results obtained by Feldtkeller, the output valve anode current-grid voltage characteristic is assumed to be an infinite power series, and expressions are obtained for the equation of the modified characteristic obtained by the application of negative feedback.

If we assume the valve characteristic to be of the form $I_a = a_1 e_g + a_2 e_g^2$, a fairly simple solution can be found.

If current feedback is used, for example, let the value of the feedback resistance be R.

$$\therefore e_g = (e_{g1} - I_a R)$$

where e_{g1} = signal input voltage.

e_g = voltage appearing between grid and cathode.

$$\therefore I_a = a_1(e_{g1} - I_a R) + a_2(e_{g1} - I_a R)^2 \quad \ldots \quad (45)$$

Solving (45) as a quadratic in I_a we get

$$I_a = \frac{1 + a_1 R + 2a_2 R e_{g1} \pm \sqrt{(1 + a_1 R)^2 + 4a_2 R e_{g1}}}{2a_2 R^2} \quad \ldots \quad \ldots \quad (46)$$

Expanding this as a series in e_{g1} and using the symbols $K_1 = 1 + a_1 R$ and $K_2 = 2a_2 R$, we get

$$I_a = \frac{K_1}{K_2 R}\left[\frac{K_2(K_1 - 1)}{K_1^2} e_{g1} + \frac{K_2^2}{2K_1^4} e_{g1}^2 - \frac{K_2^3}{2K_1^6} e_{g1}^3 + \frac{5K_2^4}{16K_1^8} e_{g1}^4 + \ldots\ldots\right] \ldots \quad (47)$$

Equation (47) gives us the new relationship existing between output current and signal voltage, when negative feedback is applied.

For a given value of fundamental output current we can find the necessary value of input signal and the percentage amplitude of the various harmonics occurring in I_a.

Without feedback, let the input signal voltage be $e_g = e \cos \omega t$ and with feedback let the input signal voltage to give the same fundamental output current be $e_{g1} = e_1 \cos \omega t$.

$$\therefore I_{a_{\omega t}} = a_1 e \cos \omega t \text{ without feedback.}$$

Also $I_{a_{\omega t}} = \frac{(K_1 - 1)e_1 \cos \omega t}{RK_1}$ from (47) with feedback.

$$\therefore a_1 e = \frac{a_1 R e_1}{R(1 + a_1 R)} = \frac{a_1 e_1}{1 + a_1 R}$$

$$\therefore \frac{e_1}{e} = (1 + a_1 R) \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

Thus for the same fundamental output current the input voltage with feedback is $(1 + a_1 R)$ times as great as the input voltage without feedback.

Let us now compare the distortions in the two cases, assuming the same fundamental output current.

Without feedback.

$$I_a = a_1 e \cos \omega t + a_2 e^2 \cos^2 \omega t$$

$$= a_1 e \cos \omega t + \frac{a_2}{2} e^2 (1 + \cos 2\omega t) \quad \ldots \quad (49)$$

$$\frac{\text{Amplitude of } \cos 2\omega t}{\text{Amplitude of } \cos \omega t} = \frac{a_2 e}{2a_1} \quad \ldots \quad \ldots \quad (50)$$

With feedback.

$$I_a = \frac{K_1}{K_2 R}\left[\frac{K_2(K_1 - 1)}{K_1^2} e_1 \cos \omega t + \frac{K_2^2}{2K_1^4} e_1^2 \cos^2 \omega t - \frac{K_2^3}{2K_1^6} e_1^3 \cos^3 \omega t + \frac{5K_2^4}{16K_1^8} e_1^4 \cos^4 \omega t \ \ldots\right]$$

$$\therefore I_a = \frac{K_1}{K_2 R}\left[\left\{\frac{K_2^2}{4K_1^4} e_1^2 + \frac{15K_2^4}{128K_1^8} e_1^4 + \ldots\right\}\right.$$

$$+ \cos\omega t\left\{\frac{K_2(K_1-1)}{K_1^2}e_1 - \frac{3K_2^3}{8K_1^6}e_1^3 + \ldots,\right\}$$
$$+ \cos 2\,\omega t\left\{\frac{K_2^2}{4K_1^4}e_1^2 + \frac{5K_2^4}{32K_1^8}e_1^4 + \ldots\right\}$$
$$+ \cos 3\omega t\ \left\{-\frac{K_2^3}{8K_1^6}e_1^3 + \ldots\right\}$$
$$+ \cos 4\omega t\ \left\{\frac{5K_2^4}{128K_1^8}e_1^4 + \ldots\right\}$$
$$+ \ldots\ldots\Big] \quad \ldots \quad (51)$$

The coefficient of $\cos\omega t$ may quite justifiably be taken as the first bracketed term only, viz. $\frac{K_2(K_1-1)}{K_1^2}e_1$

$$\text{Thus } \frac{\text{Amplitude of } \cos 2\omega t}{\text{Amplitude of } \cos\omega t} = \frac{\left\{\frac{K_2^2}{4K_1^4}e_1^2 + \frac{5K_2^4}{32K_1^8}e_1^4\right\}}{\frac{K_2(K_1-1)\,e_1}{K_1^2}}$$

$$= \frac{K_2 e_1}{4K_1^2(K_1-1)}\left[1 + \frac{5K_2^2 e_1^2}{8K_1^4}\right] \quad \ldots \quad (52)$$

For the moment, let us ignore the right-hand expression in the bracket.

$$\frac{\text{Amplitude of } \cos 2\omega t}{\text{Amplitude of } \cos\omega t} = \frac{2a_2 R e_1}{4a_1 R(1 + a_1 R)^2}$$

and since $e_1 = e(1 + a_1 R)$

$$\therefore \quad \text{Ratio} = \frac{a_2 e}{2a_1(1 + a_1 R)} \quad \ldots \quad (53)$$

Comparing this expression with (50), we see that for the same fundamental output current, the second harmonic has been reduced by a factor $(1 + a_1 R)$, i.e. the same amount as the reduction in amplification.

If we now include the second term in the bracket of equation (52) we are really including the effect of secondary distortion. The relative magnitudes of these quantities will best be appreciated by a typical example.

Let $a_1 = .01, \qquad a_2 = .002,$

$e_g = 1 \cos\omega t$ volts, $R = 900\Omega$

Plotted in the table below are the various harmonic percentages for the same fundamental output current, with and without feedback.

The results obtained on the assumption of no secondary distortion are also included.

	Per cent. Harmonic.		
Frequency Term.	Without Feedback.	With Feedback and neglecting secondary distortion.	With Feedback and allowing partly for secondary distortion.
$\cos 2\,wt$	10%	1%	1.081%
$\cos 3\,wt$	0	0	.18%
$\cos 4\,wt$	0	0	.02%

We see from this table that including the effect of secondary distortion gives a slight increase in the percentage of second harmonic, and also shows the presence of small third and fourth harmonics.

We may conclude therefore that even when an amplifier is loaded to a point which would produce 10 per cent. 2nd harmonic without feedback applied, the application of negative feedback reduces this percentage almost exactly in the same proportion that the amplification is reduced.

REFERENCES

1. "Stabilised Feedback Amplifiers," H. S. Black, *Bell S. Tech. Journ.*, January, 1934.
2. "Regeneration Theory," H. Nyquist, *Bell S. Tech. Journ.*, July, 1932.
3. "Regeneration Theory and Experiment," Peterson, Kreer and Ware, *Bell S. Tech. Journ.*, October, 1934.
4. "Verfahren zur Stabilisierung negativ rückgoppelter hochlinearer Verstärker," H. Werrmann, *V.D.E.—Fachberichten.* 1937.
5. "Resistance-Capacity coupled amplifier design," D. G. C. Luck, *Proc. Inst. Rad. Eng.*, August, 1932.
6. British Patent No. 453,246, L. I. Farren and G.E.C., Ltd.
7. British Patent No. 374,130, H. Nyquist and S.T. & C.
8. British Patent No. 443,684, H. S. Black and S.T. & C.
9. American Patent, No. 2,011,566, H. S. Black and Bell Tel. Labs.
10. R. Feldtkeller, *Teleg. u. Fernsprech. Technic.*, Vol. 25, p. 217.
11. "A Note on the Alternative Equivalent Circuit for the Thermionic Valve," N. R. Bligh, *Wireless Engineer*, September, 1930.
12. "Inverse Feedback, its application to receivers and amplifiers," B. D. H. Tellegen and V. Cohen Henriquez, *Wireless Engineer*, Aug. 1937, Vol. 14, p. 409.
13. "The necessary conditions for Instability (or Self-Oscillation) of Electrical Circuits," D. G. Reid, *Wireless Engineer*, Nov. 1937, Vol. 14, p. 588.
14. "Regeneration in Linear Amplifiers," G. S. Brayshaw *Wireless Engineer*, Nov. 1937, Vol. 14, p. 597.
15. "Wege zur Verzerrungsminderung bei Leitungsverstärkern für Mehrfach-Nachtrichtensysteme," H. Werrmann, *Telefunken*. Vol. 18, 1937, No. 77.

WE have received a bound volume, containing a number of articles describing the work of Nikola Tesla and the speeches in many languages delivered at the gathering held at Belgrade from 26th to 31st May, 1936, in celebration of Tesla's 80th birthday. Reference is also made to a number of other commemorative speeches at various centres including Paris, Vienna, Graz, Poitiers, Prague, Sofia and Brünn, where his birthday was celebrated. The volume contains 520 pages and can be obtained from the Secretary of the Institut Nikola Tesla, rue Mirocka 4, Belgrade; price 100 French Francs.