indicate that the second harmonic content is greater for frequencies near that of the bass resonance, where also the output is considerable. Thus the output at double the bass frequency is considerably augmented by distortion products. This type of harmonic production is an important factor in loud speaker quality and colours the reproduction of speech and music. The indication of this coloration given by response curves obtained by the noise analysis method is one of its advantages.

Fourthly, the response curves obtained by the two methods differ in the neighbourhood of the high frequency cut-off of the loud speaker, the noise analysis method giving a cut-off lower in frequency. This could be accounted for by a discrepancy between the frequency calibrations of the heterodyne oscillator used in the steady state measurements and the wave analyser used in the noise analysis method. These frequency calibrations have been carefully checked, however, and show no such discrepancy. Moreover there is no evidence in the response curves of a simple frequency drift at frequencies other than that of the cut-off. In all the figures shown, and in all other measurements made, this discrepancy between the curves in the region of the high frequency cut-off appears to be associated, not with a particular value of frequency, but with that frequency at which the response of the loud speaker finally falls off. Moreover the discrepancy seems to increase with the sharpness of cut-off. It should be noted that harmonic distortion and cross-modulation in the loud speaker would be expected to produce the opposite effect, giving a higher frequency of cut-off by the noise analysis method.

It is of interest to decide whether this effect is characteristic of the loud speakers in question when reproducing this type of input, or whether it is a defect in the apparatus and method of measurement, in which latter case it would appear in all response curves taken by this method. It has been suggested that the discrepancy is a function of the relative sharpness of the cut-off in the loud speaker response curves and of the response curve of the analyser-filter, that is, that the effect is inherent in the apparatus. The fact that the discrepancy does not occur at the low frequency cut-off, however, throws some doubt on this explanation. It is quite conceivable that the effect is, in fact, characteristic of the loud speaker's frequency-response, in so far as the sharp high frequency cut-off modifies the reproduction of transients.

In conclusion, the advantages of this method appear to be that, firstly, it gives a better indication of the performance of the loud speaker in normal use, by including the effects of harmonic, cross-modulation and transient distortion. Secondly, it enables response curves to be obtained in almost any surroundings whilst preserving the full detail in the loud speaker response curve, which normally is masked by the devices employed to remove the irregularities due to room effects. This method affords no saving in apparatus, for whilst a heterodyne oscillator and warbling device are dispensed with, a noise source and selective analyser are required.

Correspondence

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Distortion in Negative Feedback Amplifiers.

To the Editor, The Wireless Engineer.

SIR,—On pages 259, 369 and 607 of Vol. XIV of *The Wireless Engineer*, letters were published from Mr. Robt. W. Sloane and myself on negative feedback. Subsequently we corresponded with each other on the subject.

I have now revised my calculations on the lines indicated by Mr. Sloane in his letter to me and see that the discrepancy between our results is not due to the difference between the corresponding terms of the Taylor's and the Fourier's series as I stated in my former letter, but to my having neglected the cross modulation which will, in fact, occur between the different harmonics if they are fed back in an amplifier with distortion. I must therefore fully admit the possibility of the third, fifth or any other harmonic being increased by negative feedback.

On the other hand, as this increase is due only to the back-fed harmonics which themselves are substantially minor to the fundamental, and, as this secondary distortion is diminished, as well as the primary, proportionally with the applied degeneration, I think that under normal conditions negative feedback will not spoil the performance of the amplifier to which it is applied.

For example, let us compare the content of harmonics at the output of an amplifier, both before and after the application of negative feedback, which amplifies an input swing δe without phase distortion to an output

$$\delta E = \left[\frac{dE}{de}\right]_0 \delta e + \tfrac{1}{2}\left[\frac{d^2E}{de^2}\right]_0 \delta e^2 + \tfrac{1}{6}\left[\frac{d^3E}{de^3}\right]_0 \delta e^3$$

If we apply to this amplifier a pure sinewave

$$\delta e = A \sin \omega t$$

we will get at the output :

$$E = \left[\frac{dE}{de}\right]_0 A \sin \omega t + \tfrac{1}{2}\left[\frac{d^2E}{de^2}\right]_0 A^2 \sin^2 \omega t + \tfrac{1}{6}\left[\frac{d^3E}{de^3}\right]_0 A^3 \sin^3 \omega t$$

$$= \left[\frac{dE}{de}\right]_0 A \sin \omega t + \tfrac{1}{4}(1 - \cos 2\omega t) A^2 \left[\frac{d^2E}{de^2}\right]_0 + \frac{1}{24}(3 \sin \omega t - \sin 3\omega t) A^3 \left[\frac{d^3E}{de^3}\right]_0$$

$$= \tfrac{1}{4}A^2\left[\frac{d^2E}{de^2}\right]_0 + \left\{\left[\frac{dE}{de}\right]_0 A + \tfrac{1}{8}A^3\left[\frac{d^3E}{de^3}\right]_0\right\}\sin \omega t - \tfrac{1}{4}A^2\left[\frac{d^2E}{de^2}\right]_0 \cos 2\omega t - \frac{1}{24}A^3\left[\frac{d^3E}{de^3}\right]_0 \sin 3\omega t$$

The proportion of the harmonics will thus be :

$$K_2 = \frac{\tfrac{1}{4}A^2\left[\frac{d^2E}{de^2}\right]_0}{A\left[\frac{dE}{de}\right]_0 + \tfrac{1}{8}A^3\left[\frac{d^3E}{de^3}\right]_0} \simeq \frac{\tfrac{1}{4}A^2\left[\frac{d^2E}{de^2}\right]_0}{A\left[\frac{dE}{de}\right]_0} = \frac{A}{4}\frac{\left[\frac{d^2E}{de^2}\right]_0}{\left[\frac{dE}{de}\right]_0}$$

$$K_3 = \frac{\frac{1}{24}A^3\left[\frac{d^3E}{de^3}\right]_0}{A\left[\frac{dE}{de}\right]_0 + \tfrac{1}{8}A^3\left[\frac{d^3E}{de^3}\right]_0} \simeq \frac{A^2}{24}\frac{\left[\frac{d^3E}{de^3}\right]_0}{\left[\frac{dE}{de}\right]_0}$$

or inversely :

$$\left[\frac{d^2E}{de^2}\right]_0 \simeq \frac{4}{A}\left[\frac{dE}{de}\right]_0 K_2$$

$$\left[\frac{d^3E}{de^3}\right]_0 \simeq \frac{24}{A^2}\left[\frac{dE}{de}\right]_0 K_3$$

If we now apply a fraction β of the output at the input as degeneration, we have for the new amplifier according to Mr. Sloane's calculation :

$$\left[\frac{dE}{de}\right]_0^* = \frac{\left[\frac{dE}{de}\right]_0}{1 + \beta\left[\frac{dE}{de}\right]_0}$$

$$\left[\frac{d^3E}{de^3}\right]_0^* = \frac{\left[\frac{d^3E}{de^3}\right]_0}{\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^4} - \frac{3\left[\frac{d^2E}{de^2}\right]_0^2 \beta}{\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^5}$$

if we now increase the input from $A \sin \omega t$ to $A^* \sin \omega t = A\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)\sin \omega t$ in order to get the same output as before, the proportion of the third harmonic will be with the same approximation as above

$$K_3^* \simeq \frac{A^{*2}}{24}\frac{\left[\frac{d^3E}{de^3}\right]_0^*}{\left[\frac{dE}{de}\right]_0^*} =$$

$$= \left[\frac{A^2\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^2}{24} \cdot \frac{\left[\frac{d^3E}{de^3}\right]_0}{\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^4} \cdot \frac{A + \beta\left[\frac{dE}{de}\right]_0}{\left[\frac{dE}{de}\right]_0}\right] - \left[\frac{3\left[\frac{d^2E}{de^2}\right]_0^2 \beta}{\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^5} \cdot \frac{A^2\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^2}{24} \cdot \frac{1 + \beta\left[\frac{dE}{de}\right]_0}{\left[\frac{dE}{de}\right]_0}\right]$$

$$= \frac{A^2}{24} \cdot \frac{\left[\frac{d^3E}{dE^3}\right]_0}{\left[\frac{dE}{de}\right]_0} \cdot \frac{1}{1 + \beta\left[\frac{dE}{de}\right]_0} - \frac{A^2}{8} \cdot \frac{\left[\frac{d^2E}{de^2}\right]_0^2}{\left[\frac{dE}{de}\right]_0} \cdot \frac{\beta}{\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^2}$$

If we express this in terms of the proportion of harmonics without degeneration, we get

$$K_3^* \simeq \frac{K_3}{1 + \beta\left[\frac{dE}{de}\right]_0} - 2\,\frac{\beta\left[\frac{dE}{de}\right]_0}{\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^2} K_2^2.$$

We see that whilst the initial proportion of the third harmonic will decrease by $1 + \beta\left[\frac{dE}{de}\right]_0$, there will appear, according to Mr. Sloane's statement, another part.

For a given K_2 this second part will be 0 for $\beta = 0$ and for $\beta =$. It will have its maximum at $\beta\left[\frac{dE}{de}\right]_0 = 1$, in which case

$$2\,\frac{\beta\left[\frac{dE}{de}\right]_0}{\left(1 + \beta\left[\frac{dE}{de}\right]_0\right)^2} = 2\,\frac{1}{2^2} = \tfrac{1}{2}$$

and the additional part is $\tfrac{1}{2} K_2^2$. For any other value of β it will be less. If we now take for the initial second harmonic of the amplifier without feedback the extremely high value of $K_2 = 20$ per cent., the additional part of third harmonic will be $\tfrac{1}{2}K_2^2 = \tfrac{1}{2}\,0.2^2 = 2$ per cent. For $K_2 = 10$ per cent., it will be $\tfrac{1}{2}\,0.1^2 = 0.5$ per cent.

If we now consider that an amplifier which generates such a high proportion of second harmonics, will seldom be quite free from third harmonics and that the initial harmonics are diminished in proportion to the applied degeneration, it is hard to imagine a practical case in which negative feedback would do more harm than good.

These practical observations do not, of course, affect the value of Mr. Sloane's statement, which I denied in my first letter, and I am much indebted to him for having pointed out my error.

In conclusion it can be stated that: negative feedback diminishes all harmonics approximately in the proportion in which the sensitivity of the amplifier is diminished, but owing to distortion and crossmodulation of the back-fed harmonics, this relation is not exact. Fortunately in ordinary amplifiers the difference is only negligible.

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To the Editor, The Wireless Engineer.

SIR,—Considering the literature which has been written on negative feedback, some of which appeared recently in your esteemed journal[1, 2, 3] I should like to point out some facts which as far as I know have been neglected.

The negative feedback amplifier was treated alone without considering the harmonic distortion of the preceding valve amplifier. The application of negative feedback to a valve circuit decreases the amplification of the valve. Therefore to obtain the same output from the valve with feedback as without, we must increase the amplification of the preceding stage—the corresponding increase in the harmonic distortion may exceed the decrease obtained by providing negative feedback, so that the total harmonic distortion may be increased.

Let e be the e.m.f. of the input of the preceding stage, u, the p.d. of the output of the same, U, the p.d. of the output of the final stage, again e', u', V', the correspondong values for the negative feedback case. Using the conventional method of deriving u, from e, V, from u, we may write :

$$u = a_1 e + a_2 e^2 +$$
$$U = \alpha_1 u + \alpha_2 u^2 + \ \ldots = \alpha_1 a_1 e + (\alpha_2 a_1^2 + \alpha_1 a_2)e^2 + \ \ldots$$

and for the negative feedback case :

$$u' = a_1 e' + a_2 e'^2 + \ \ldots$$

$$V' = \frac{\alpha_1}{1 + \alpha_1\beta} u' + \frac{\alpha_2}{1 + \alpha_1\beta} u'^2 + \ \ldots$$
$$= \frac{\alpha_1 a_2}{1 + \alpha_1\beta} e' + \left(\frac{\alpha_1 a_2}{1 + \alpha_1\beta} + \frac{\alpha_2 a_1^2}{1 + \alpha_1\beta}\right) e'^2 + \ \ldots$$

If the output (without harmonics) is to be the same in both cases : $e' = (1 + \alpha_1\beta)e$; therefore :

$$V' = \alpha_1 a_1 e + \left(\alpha_1 a_2(1 + \alpha_1\beta) + \frac{\alpha_2 a_1^2}{1 + \alpha_1\beta}\right) e^2 + \ \ldots$$

To take advantage, as far as harmonic distortion is concerned, from negative feedback, we must have:

$$\alpha_1 a_2(1 + \alpha_1\beta) + \frac{\alpha_2 a_1^2}{1 + \alpha_1\beta} < \alpha_2 a_1^2 + \alpha_1 a_2$$

finally :

$$\frac{\alpha_2}{\alpha_1} > \frac{a_2}{a_1} \Big/ \frac{a_1}{1 + \alpha_1\beta};$$

Beyond a certain limit given by the above relation, negative feedback no longer improves the harmonic distortion of the system. I cannot discuss here in more detail this relation. It would be interesting that such curves as those given in previous works[2, 4] giving harmonic distortion of current against output watts, with and without negative feedback, should be traced taking into account the total harmonic distortion as explained above.

I must point out here that in some previous works of mine[5, 6, 7] concerning a non-linear positive feedback, I have obtained better results than those given in the references mentioned above, results that are not restricted by the above considerations. I also have obtained[8] very good results concerning frequency distortion, with positive feedback (negative impedance method).

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1. *Wireless Engineer*, Vol. XIV, p. 259.
2. *Wireless Engineer*, Vol. XIV, p. 409.
3. *Wireless Engineer*, Vol. XIV, p. 597.
4. *Philips' Technique Rundschau*, Sept., 1936, p. 269.
5. *L'onde Electrique*, Vol. XV, p. 469.
6. *Wireless Engineer*, Vol. XIII, p. 131.
7. *Comptes Rendus*, Vol. 201, p. 193.
8. *Wireless Engineer*, Vol. XII, p. 375.

The Battery Book

By H. H. U. Cross. XII + 196 pp. 92 Figs. The Technical Press, Ltd., 5, Ave Maria Lane, London, E.C.4. Price 5/-.

This is an excellent little book. Its subtitle is a Practical Manual on the construction, charging, care and repair of automobile, motor cycle, aviation, electric vehicle, medical and other similar batteries, and there is a foreword by Mr. J. Y. Fletcher, a Director of the General Electric Co. The author states in the preface that the book is written from the popular angle, but that scattered throughout the pages will be found a measure of scientific information, so stated that the general reader may assimilate it without effort. The arrangement of the material is based upon the author's book in French on the same subject which was recently published in France and Belgium. The first chapter deals with dry and other primary batteries and the subsequent chapters deal with various types of cells, lead, zinc, halogen and alkaline, their construction, operation and applications. The author is evidently an authority both on the practical and theoretical sides of the subject, and the material is arranged and presented in a very readable form. It is a book which may be recommended not only to all those who are primarily interested in accumulators, but also to every student of electrical engineering. G. W. O. H.