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Feedback models reduce op-amp circuits to voltage dividers

An op amp's feedback factor defines a range of performance characteristics. Unfortunately, this factor is unknown for most op-amp applications because of a limited feedback model. By extending this model ,ou can create a generalized feedback model :hat reduces op-amp circuit analysis to determining voltage-divider ratios.

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The feedback factor of an op-amp circuit defines that circuit's performance more than any other parameter. The feedback factor sets the gain of the op amp's input-referred errors. These errors include offset voltage, noise, and the error signals generated by limited open-loop gain, common-mode rejection, and power-supply rejection. In addition, a circuit's feedback factor determines bandwidth and frequency stability. Yet this powerful performance indicator remains unknown for most op-amp applications. Except for the basic noninverting op-amp connection, the classic feedback model does not predict the feedback factors of op-amp circuits.

In the noninverting case, the closed-loop gain relates directly to the feedback factor; the application gain itself determines the output errors and bandwidth. However, the relationship between the gain and feedback factor does not extend to other op-amp configurations. In other configurations, several conditions make the gain-feedback relationship unclear. The input and output signals of inverting op-amp connections, for example, combine on the feedback network to conceal the feedback factor. Other applications have both positive and negative feedback, which results in more than one feedback factor. In still other applications, bootstrap feedback adds another variable that the classic feedback model does not take into account. Without knowing the feedback factor, you must perform laborious calculations to determine these circuits' performance.

You can, however, extend the convenience of a feedback factor to these other circuits by modifying the classic op-amp feedback model. Specific connection examples can demonstrate the possible variations to this model. These variations are limited in number by the two inputs of an op amp; you can connect the input and feedback signals of an op amp in only a few ways. The examples in **Figs 1** through **11** demonstrate modeling principles that will let you create a feedback model for any op-amp configuration. The final example is a universal op-amp feedback model that has standardized performance equations.

For the noninverting op-amp configuration, a direct relationship between the closed-loop gain and the feedback factor simplifies analyzing circuit performance.

The feedback factor of an op amp defines the circuit performance more than any other parameter.

Fig 1 shows this configuration as a voltage amplifier. This noninverting circuit provides the familiar, **ideal/closed-loop gain (A_{CLI}): A_{CLI} = ($R_1 + R_2$)/R_1.** This gain amplifies both the input signal (e_I) and the differential input errors (e_{ID}) of the op amp. Multiplying the input-referred amplifier errors by A_{CLI} yields the resulting output errors.

As you can see in the **Fig 1** model, the mechanism relating both the input and output errors is the feedback factor. This model represents the noninverting op-amp connection by an amplifier with differential-input-error signal e_{ID} and feedback factor β. This feedback factor defines the portion of the output signal (e_O) that feeds back to the amplifier input. Writing a loop equation for this model shows that $e_O = (1/\beta)(e_I - e_{ID})$. Thus, the feedback model shows that 1/β—rather than A_{CLI}—amplifies e_I and e_{ID}.

To resolve this amplification difference, define the noninverting amplifier's feedback factor. The feedback factor is the fraction of the amplifier's output that feeds back to its input. In **Fig 1,** the voltage-divider action of the feedback resistors sets the fraction of e_O fed back to the op-amp input: $\beta e_O = e_O R_1/(R_1 + R_2)$.

This relationship defines β as the voltage-divider ratio of the feedback network. Comparing this result with the A_{CLI} expression shows that $A_{CLI} = 1/\beta$ for the noninverting op-amp configuration. Thus, the circuit and model are in agreement for the input-to-output transmission of amplifier errors.

The types of amplifier errors this model takes into account are numerous because e_{ID} includes errors related to several amplifier characteristics. Each of these characteristics produces an input-referred error source for the op amp. The following formula represents error sources related to op-amp input-offset voltage (V_{OS}), input-noise voltage (e_N), open-loop gain (A), common-mode rejection ratio (CMRR), and power-supply rejection ratio (PSSR). The last three error terms include circuit signals: the output voltage (e_O), the common-mode voltage (e_{ICM}), and the change in power-supply voltage (δV_S):

$$e_{ID} = V_{OS} + e_N + (e_O/A) + (e_{ICM}/CMRR) + (\delta V_S/PSSR).$$

To find the amplifier output errors each of these terms creates, multiply each term by the 1/β factor of the application circuit. Some familiar error terms result from this multiplication. The output error due to the finite open-loop gain becomes $e_O/A\beta$, which shows that the output signal is diminished by a fraction equal to the reciprocal of loop gain Aβ. The decline of open-loop-gain A with frequency makes this output error rise, thus shaping the closed-loop frequency response of the circuit. The output-noise error term is e_N/β, leading to the term "noise gain" for 1/β. This description of 1/β is accurate only under certain bandwidth limits. For both the loop-gain and noise errors, greater insight into circuit performance results from frequency-response analysis.

For the noninverting circuit in **Fig 1,** the multiplier

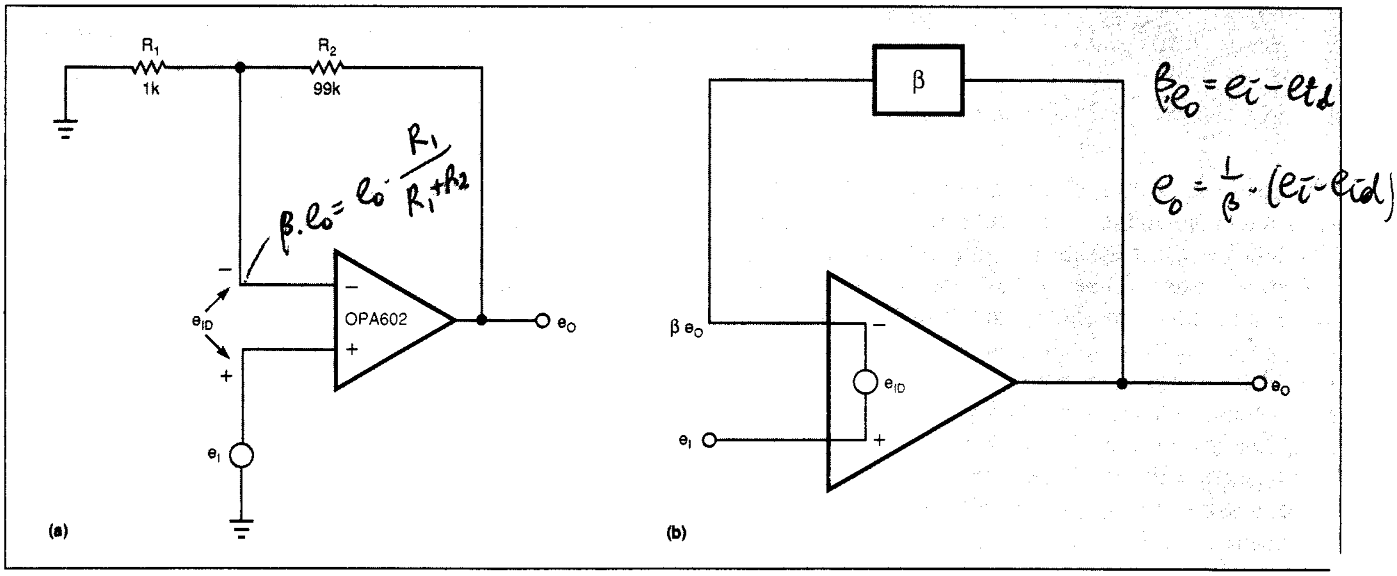

***Fig 1— Op-amp input errors** of the circuit schematic **(a)** are amplified by the reciprocal of the feedback factor, 1/β, in the model **(b)**.*

that relates the input and output errors conveniently equals A_{CLI}. Other op-amp configurations do not share this convenience. For these configurations, you must determine the $1/\beta$ factor independently of the ideal closed-loop gain. Once you determine this factor, the error-analysis process is the same as that of the **Fig 1** circuit.

For these more-complex op-amp configurations, you need to use feedback modeling to determine the feedback factor. This modeling also yields frequency-response and frequency-stability information. To demonstrate modeling, consider the familiar noninverting circuit in **Fig 2.** This noninverting configuration highlights the voltage-divider action of the feedback network. For more general use, the network has impedances Z_1 and Z_2 rather than the resistors in **Fig 1.** As before, the network's divider action controls the fraction of the amplifier output fed back to the amplifier input. The **Fig 2** circuit reduces input-error-signal e_{ID} to the value of the open-loop gain error, e_O/A. This reduction is due to the fact that the feedback modeling focuses only on gain and related frequency characteristics. Nevertheless, the one input-referred error is sufficient to define the feedback factor for use with the previous multi-error analysis.

Fig 2 also shows the feedback model for the noninverting op-amp connection. This classic feedback model, initially developed by Black **(Ref 1)**, is generally proposed for op-amp circuits. However, this model applies only to the noninverting case and needs modification for other configurations. The model represents amplifier gain by gain-block A. A summation block, Σ, drives the inputs of the gain block. The summation block's inputs are input-signal e_I and feedback-signal βe_O. The feedback signal flows through feedback-attenuator block β. The summation block applies different polarities to the two signals, as the + and − signs indicate. These polarities correspond to the amplifier-input polarities of the actual circuit.

You can demonstrate the validity of the model by comparing the closed-loop-gain (A_{CL}) responses for the model and the circuit. For the model, the output signal is $e_O = A(e_I - \beta e_O)$. Solving this equation for e_O/e_I defines the modeled transfer response of the noninverting circuit as

$$A_{CL} = e_O/e_I = A/(1 + A\beta).$$

For the actual circuit of **Fig 2**, the transfer response of a noninverting amplifier is

$$A_{CL} = \frac{e_O}{e_I} = \frac{A}{1 + \frac{AZ_1}{Z_1 + Z_2}}.$$

Comparing the terms in the last two equations shows that the feedback factor is $\beta = Z_1/(Z_1 + Z_2)$. $Z_2/(Z_1+Z_2)$

The preceding analysis confirms the accuracy of the

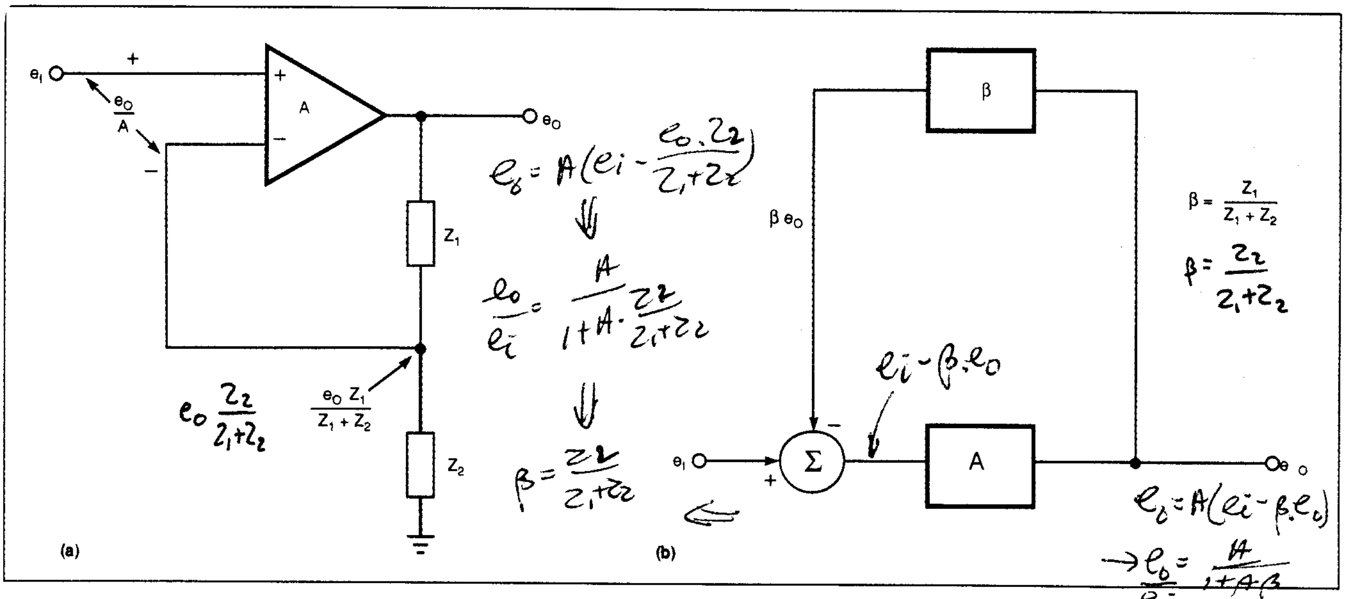

Fig 2—A comparison of circuit (a) and model (b) responses shows that the classic feedback model predicts op-amp performance in noninverting connections.

Except for the basic noninverting case, the classic feedback model does not predict the feedback factor of op-amp circuits.

Fig 2 model and provides the basis for determining the frequency response and stability of the circuit. This added performance information is based on the feedback factor and is not specific to the noninverting case. Using feedback modeling, you can derive the frequency characteristics of an op-amp circuit by analyzing the model's closed-loop-response equation (**Ref 2**). For the noninverting case, you can rewrite this equation as

$$A_{CL}=\frac{1/\beta}{1+1/A\beta}. \quad (1)$$

Other op-amp configurations have different closed-loop-response equations, but these equations always have the same $1+1/A\beta$ denominator. This common denominator is central to the bandwidth and stability characteristics that hold for all op-amp configurations.

The frequency response of the **Fig 2** circuit begins with the value of the ideal closed-loop gain (A_{CLI}) at dc. Because the op-amp open-loop gain (A) is very high at dc, the previous closed-loop-response equation simplifies to the ideal gain of the noninverting circuit: $A_{CLI}=1/\beta$. At higher frequencies, the op-amp open-loop gain declines, causing the closed-loop gain to drop from the ideal value. This drop produces the circuit's bandwidth limit, as shown in **Fig 3**, which is a plot of the op amp's closed-loop response, its open-loop gain, and the reciprocal of the feedback factor. All three variables of the original closed-loop-response equation are plotted on the same graph. The manner in which these variables interact in **Fig 3** provides visual insight into bandwidth and frequency-stability limits.

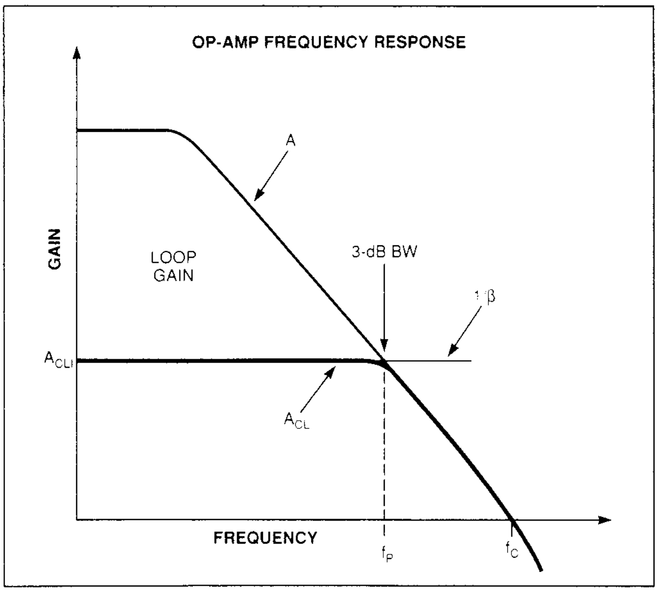

***Fig 3—The feedback factor indicates op-amp bandwidth and stability** through the relationship between the 1/β curve and the open-loop-gain curve, A.*

The circuit-loop gain, $A\beta$, represents the amplifier gain resource available to maintain the ideal closed-loop response. In **Fig 3**, the separation between the A and 1/β curves represents the loop gain. Because of the logarithmic scale of response plots, this separation equals log (A) − log (1/β), which equals log (Aβ). Thus, at any given frequency, loop-gain Aβ is the vertical distance between the A and 1/β curves. Where the loop gain can no longer match the feedback demand, the closed-loop curve drops from the ideal A_{CLI}. The A and 1/β curves graphically define this point. The 1/β curve represents the feedback demand, and ideal closed-loop requirements are met as long as 1/β is below the open-loop-gain curve. Where this condition is no longer true, the actual response drops and follows the amplifier open-loop response downward. The rate of descent for the roll-off is −20 dB/decade for most op amps, a slope that is characteristic of a single-pole response. The heavier curve in **Fig 3** represents the resulting closed-loop gain, A_{CL}.

The location of pole f_P in the A_{CL} response roll-off determines the closed-loop bandwidth of the circuit. At the pole frequency, A_{CL} drops from its dc level of 1/β to 0.707(1/β). This drop assumes that resistor feedback produces a frequency-independent β. Under this condition, the gain drop occurs at the intercept frequency of the A and 1/β curves. These curves are actually magnitude response curves, and, at their intercept, $|A|=|1/\beta|$ or $|A\beta|=1$. The single-pole roll-off of gain A develops a phase of −90°. Thus, $A\beta=-j1$ at the intercept, and the denominator of **Eq 1** is $1+(1/A\beta)=1+j1$.

At the intercept, the magnitude of the denominator increases from its dc level of 1 to $\sqrt{2}$, and A_{CL} drops to 0.707(1/β). Thus, for frequency-independent feedback factors, the 3-dB bandwidth occurs at the intercept frequency of the A and 1/β curves. Where the feedback factor is frequency dependent, the closed-loop response still rolls off following the intercept, but this point may not be the 3-dB bandwidth limit. Peaking in the closed-loop response curve may move the actual 3-dB point away from the intercept frequency.

For more-common op-amp applications, the feedback factor is constant, and a simple equation defines the 3-dB bandwidth. The open-loop response of most op amps has a single-pole roll-off. Virtually all intercepts of the A and $1/\beta$ curves occur in this single-pole range. In this range, the gain magnitude is $A=f_C/f$, where f_C is the unity-gain crossover frequency of the amplifier. At the intercept, $f=f_P$, and $A=1/\beta=f_C/f_P$. Solving for the 3-dB bandwidth (BW) for most op-amp applications is $BW=f_P=\beta f_C$.

This result holds for all op-amp applications having frequency-independent βs and a single-pole op-amp roll-off. In other cases, you find the 3-dB response limit by considering the added phase shift caused by the increased amplifier roll-off or by a frequency-dependent feedback factor.

Knowing the A_{CL} frequency response, you can refine the simple analysis of **Fig 1** so you can apply it to broader frequency ranges. The previous analysis showed that input-referred errors of op amps transfer to the amplifier output through a gain of $A_{CLI}=1/\beta$. However, both A_{CLI} and $1/\beta$ are independent of the amplifier's high-frequency limitation. The **Fig 1** analysis is valid only when the op amp has sufficient gain to support the feedback demand. The 3-dB bandwidth marks a response roll-off that reduces amplification of both the signal and the error. Thus, op-amp error signals receive a gain of $A_{CLI}=1/\beta$ only to the frequency where $BW=\beta f_C$. Beyond this limit, the gain available to error signals rolls off and follows the op-amp open-loop response in **Fig 3**.

This roll-off produces the difference between $1/\beta$ and the noise gain. The noise gain follows the roll-off **Fig 3** describes even though the $1/\beta$ curve continues uninterrupted. The denominator of the A_{CL} equation (**Eq 1**) expresses this roll-off. The closed-loop error gain, A_{CLE}, is

$$A_{CLE}=\frac{1/\beta}{1+1/A\beta}. \qquad (2)$$

This error gain is frequency dependent. Higher-frequency noise and CMRR and PSRR errors receive diminishing gain. Note that A_{CLE} depends on only the variables β and A. Any feedback model with β and A blocks configured as in **Fig 2** yields the same expression for A_{CLE}. This model configuration and the A_{CLE} result apply to all op-amp configurations.

Using response plots like **Fig 3**, you can evaluate the frequency stability of an op-amp circuit from the curve slopes. The slopes of the A and $1/\beta$ curves at their intercept indicate phase shift for a critical feedback condition. At this intercept, $|A\beta|=1$; a loop phase shift of 180° makes $A\beta=-1$. Then, the $1+(1/A\beta)$ denominator of **Eq 1** is zero, and A_{CL} is infinite. With infinite gain, a circuit can support an output signal in the absence of an input signal, meaning the circuit can oscillate. To prevent oscillation, you must keep the phase of Aβ below 180°. To prevent response ringing, you must further limit this phase to 135° or less.

You determine the loop phase shift by relating phase shifts to the slopes of the gain magnitude and $1/\beta$ curves. The relationship between the response slope and the phase shift is based on the effects of response poles and zeros. A pole creates a −20-dB/decade response roll-off and −90° of phase shift; a zero produces the same effects but with opposite polarities. Additional poles and zeros add response slopes and phase shifts in increments of the same magnitudes. The slope and phase correlation is an accurate approximation when the critical intercept is well separated from response-break frequencies. When the intercept is less than one decade from a response break, you have to use a more detailed phase analysis (**Ref 2**). Even in these cases, the response slopes provide insight into probable stability behavior.

Relying on the slope and phase correlation, you determine the feedback phase shift from the gain magnitude and $1/\beta$ curves. The intercept point in **Fig 3** occurs after the amplifier's first pole develops the 90° phase shift but well before the second pole has any effect. At the intercept, the gain-magnitude curve has a slope of −20 dB/decade, and the $1/\beta$ curve has zero slope for a net 90° feedback phase shift. The result leaves a phase margin of 90° from the 180° needed to cause oscillation. The zero slope of the $1/\beta$ curve in **Fig 3** is characteristic of voltage-amplifier op-amp applications. In these applications, resistors form the feedback network. In other applications, capacitors are often part of this network and effect a nonzero $1/\beta$ slope.

Inverting configuration extends model

You can define the feedback factor and closed-loop gain for less obvious op-amp configurations by extending feedback modeling. The following examples demonstrate modifications of the **Fig 2** basic feedback model that you need for alternate signal and feedback connections. In each case, the A_{CL} transfer-function

The input and output signals of inverting op-amp connections combine on the feedback network to obscure the feedback factor.

has a denominator of $1+(1/A\beta)$, and **Eq 2** describes the error-signal gain.

The first example is the simple inverting op amp **(Fig 4)**. This circuit interchanges the ground and e_I connections of **Fig 2**. This modification complicates determining the feedback factor for both the circuit and the model because the fraction of the amplifier output fed back to the input is not immediately obvious. The inverting input of the op amp is held near zero voltage by the inherent operation of an inverting circuit. This action results because the voltage at the inverting input receives counteracting signals from e_O and e_I.

The signals the op amp receives result from the voltage-divider action of the feedback network; e_O and e_I drive the divider at opposite ends. Superposition of these divider actions shows that the signal at the amplifier's inverting input or summing junction (e_{SJ}) is

$$e_{SJ}=\frac{e_O Z_1}{Z_1+Z_2}+\frac{e_I Z_2}{Z_1+Z_2}.$$

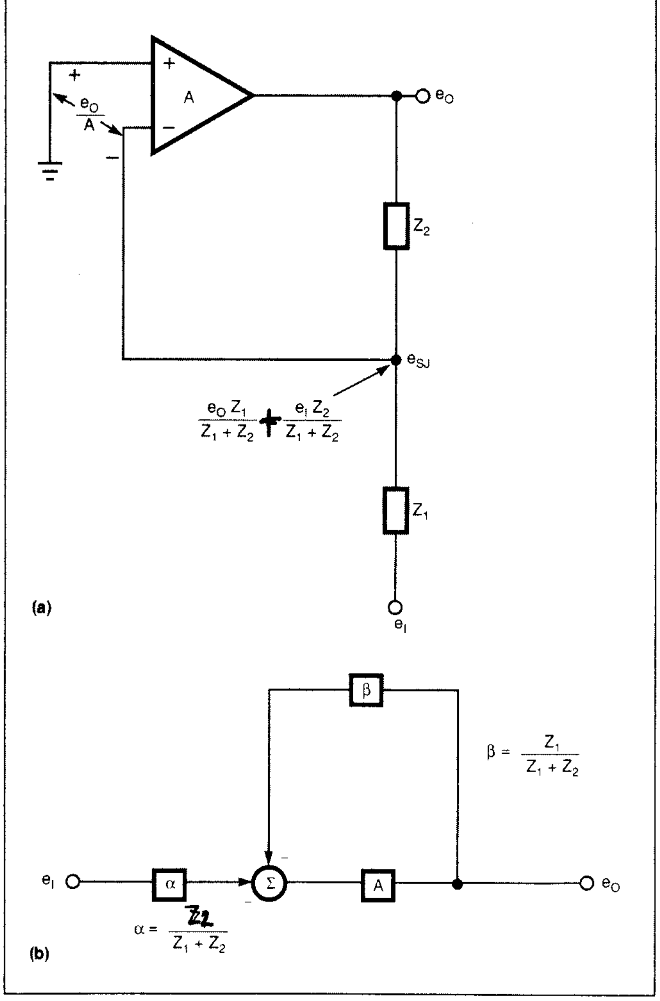

Fig 4—Inverting op-amp circuits (a) require model modifications (b) *for an input signal that is attenuated and delivered to the opposite amplifier input.*

The first term of this equation shows that $Z_1/(Z_1+Z_2)$ remains the fraction of the output fed back to the input. Thus, for op-amp feedback networks, the feedback factor is the voltage-divider ratio of the network, regardless of the signals present in the actual circuit.

Analyzing **Fig 4** with the feedback model requires you to adjust for the input-signal connection. The classic feedback model of **Fig 2** shows e_I driving a noninverting or positive input at the summation point. This arrangement corresponds to the signal connection at the amplifier's noninverting input. However, in **Fig 4**, e_I is coupled to the amplifier's inverting input rather than its noninverting input. **Fig 4** accommodates this difference by changing the sign of the corresponding summation input. In op-amp feedback modeling, assign all summation inputs the same polarities as the corresponding amplifier inputs.

Also, the **Fig 2** model shows e_I connected directly to the summation point in accordance with the direct connection of the circuit. **Fig 4**, however, shows e_I connected to the feedback network rather than directly to the amplifier input. This network attenuates the amplifier input as the equation for e_{SJ} reflects. To include this attenuation in the feedback model, **Fig 4** adds feed-forward factor α. This feed-forward factor is the fraction of the input signal fed forward to the amplifier input. As with the feedback factor, a voltage-divider ratio of the feedback network defines the feed-forward factor. For α, the divider ratio is taken from the opposite end of the feedback network. For **Fig 4**, $\alpha=Z_2/(Z_1+Z_2)$. In practice, every input signal connection to a feedback model has a corresponding α. For direct signal connections to amplifier inputs, α is unity.

Extended model simplifies inverter analysis

Using these model adjustments, you can extend feedback analysis to predicting the performance of inverting circuits. The feedback model of **Fig 4** sums the input and feedback signals for $e_O=A(-\alpha e_I-\beta e_O)$. Solving this equation for e_O/e_I yields the model response. **Fig 4** compares the model with the corresponding circuit. Comparing terms confirms the defined values of

α and β. Rewriting the model result shows that the closed-loop gain of the generalized inverting circuit is

$$A_{CL}=\frac{-\alpha/\beta}{1+1/A\beta}. \quad (3)$$

When the loop-gain $A\beta$ is large, the equation reduces to the ideal closed-loop gain of $A_{CLI}=-\alpha/\beta=-Z_2/Z_1$.

The magnitude of this closed-loop gain is lower than the $(Z_1+Z_2)/Z_1$ of the noninverting case, but the bandwidth is not correspondingly higher. This relationship results from the fact that the feedback factor—not the closed-loop gain—controls the bandwidth. The two circuits have the same feedback factor even though their gain magnitudes are different. As a result, the gain-bandwidth product drops when the circuit changes from the noninverting to the inverting configuration.

To quantify bandwidth for the inverting case, the previous noninverting analysis transfers directly. This transfer results from the standard form of the response equations. The noninverting bandwidth was derived from the denominator of the A_{CL} response (**Eq 1**). That same $1+(1/A\beta)$ denominator applies to the inverting case as **Eq 3** shows. In both cases, the A_{CLI} numerator reflects the ideal closed-loop gain. As in **Fig 2,** the bandwidth for the inverting op-amp connection is βf_C, even though the closed-loop gain has decreased.

This βf_C relationship extends to all other op-amp configurations as well. You can write the transfer response of any negative-feedback system in a form that includes the $1+(1/A\beta)$ denominator. In this form, the numerator of the response equation reflects the ideal closed-loop gain. This gain describes the transfer response when $A\beta>>1$, thus making the denominator essentially unity. The standard equation for the generalized transfer response for any op-amp configuration is

$$A_{CL}=\frac{A_{CLI}}{1+1/A\beta}.$$

Feedback modeling can reduce the transfer response of any op-amp connection to this generalized form. The conclusions you draw from this standard equation translate to all op-amp connections. Rederiving the characteristics of each individual configuration is unnecessary. The only variable factor is ideal-gain A_{CLI}, which you express in terms of α and β combinations that are unique to a given configuration. For a given configuration, you can find A_{CLI} by writing the response of the feedback model in the standard form.

You can also express the op-amp input-error gain, A_{CLE}, in a generalized form. In this case, there are no differences between the equations for different amplifier configurations. For **Fig 4,** this gain is the gain of error-signal e_O/A. This gain also affects the other input-referred error signals of e_{ID}. For the **Fig 4** circuit, you can find A_{CLE} by using superposition and a test signal. Setting e_I to zero, you add a second error signal, such as noise (e_N), to the e_O/A error signal. This procedure has the same effect as adding an e_N generator in series with the amplifier's noninverting input. The gain of this configuration amplifies such a signal, and

$$A_{CLE}=\frac{1/\beta}{1+1/A\beta}. \quad (4)$$

Thus, A_{CLE} for the inverting configuration is the same as that of the noninverting case. Further examples show this equation to be true for all configurations. Op-amp input-referred error signals are amplified by $1/\beta$ up to the response roll-off the $1+1/(A\beta)$ denominator creates. From the **Fig 3** discussion, this roll-off starts with the closed-loop bandwidth of the amplifier. Beyond this bandwidth limit, A_{CLE} follows the op amp's open-loop response.

Multiple paths extend possibilities

Other variations of op-amp configurations result from dual feedback paths or dual input-signal connections. **Fig 5** shows a configuration with feedback to both amplifier's inputs. A voltage-follower connection provides unity feedback to the inverting input, and a feedback network supplies positive feedback to the noninverting input. Normally, positive feedback degrades circuit stability, but, in the **Fig 5** example, the opposite is true. Positive feedback is useful when a greater negative feedback makes the overall circuit feedback negative. The combined feedback effects determine circuit operation, as feedback modeling illustrates.

The purpose of the dual feedback is to achieve voltage-follower operation with an op amp that is not phase compensated for unity-gain stability. Normally, a voltage follower must have unity-gain stability because of the follower's unity feedback. However, some op amps lack this stability because of reduced internal phase compensation. Numerous op amps offer different degrees of phase compensation. Often, one product ver-

You can extend the feedback-factor convenience to all op-amp circuit configurations through feedback modeling.

sion will have unity-gain stability but will also have a far slower slew rate than a lesser compensated version. The slew rate of the OPA37 in **Fig 5** is 12V/μsec, and the device's phase compensation is set for gains of five or greater. A companion product, the OPA27, has unity-gain phase compensation, but its greater compensation reduces the slew rate to 2V/μsec. Typically, the devices' slew rates differ by a factor approximately equal to the minimum gain of the lesser compensated version.

Modifying the circuit's feedback factor makes the higher slew rate available to the voltage follower. The modification reduces the feedback factor without altering the closed-loop gain, which removes the requirement for unity-gain phase compensation. The frequency stability of an op-amp configuration depends on the phase shift at the intercept of the A and 1/β curves. **Fig 5** shows these curves for the reduced phase compensation and added positive feedback of the example. Because of the reduced compensation, the open-loop-gain curve A exhibits two response poles above the unity-gain axis. As a result, the slope of this curve is −40 dB/decade when the curve reaches unity gain.

This slope corresponds to 180° of phase shift and indicates oscillation for a 1/β intercept at unity gain. Normally, this intercept would result with a voltage follower where $1/\beta = 1$. However, the positive feedback of the **Fig 5** circuit reduces the net feedback factor to raise the 1/β curve. The raised curve places the intercept in a region of reduced open-loop-gain slope and ensures frequency stability.

Raising the 1/β curve also moves the intercept back in frequency, which reduces the closed-loop bandwidth. In practice, this bandwidth reduction is the same as that produced by using the unity-gain compensated version of the amplifier as a conventional voltage follower. In that case, the added internal phase compensation reduces the bandwidth. To get the greatest bandwidth from the circuit in **Fig 5,** set the intercept at the level of the amplifier's minimum rated gain. This intercept condition results in $1/\beta = A_{MIN}$, where A_{MIN} is the minimum stable gain the manufacturer specifies for the amplifier.

To permit this feedback setting, you must determine the value of β for **Fig 5.** Once again, the feedback-factor definition and the basic feedback model fail in this task. Determining the fraction of the output fed back to the input is complicated by the dual feedback paths. The classic feedback model of **Fig 2** offers no help because it represents only one feedback path. **Fig 6** extends the **Fig 2** model to the dual-feedback circuit of **Fig 5** by incorporating two adjustments. First, the model adds feed-forward factor α in series with the signal input, following the process described for **Fig 4.** However, the model couples e_I to the positive inputs on the amplifier and summation elements.

The second model change is the addition of the β_+ feedback path, which connects to a positive input on

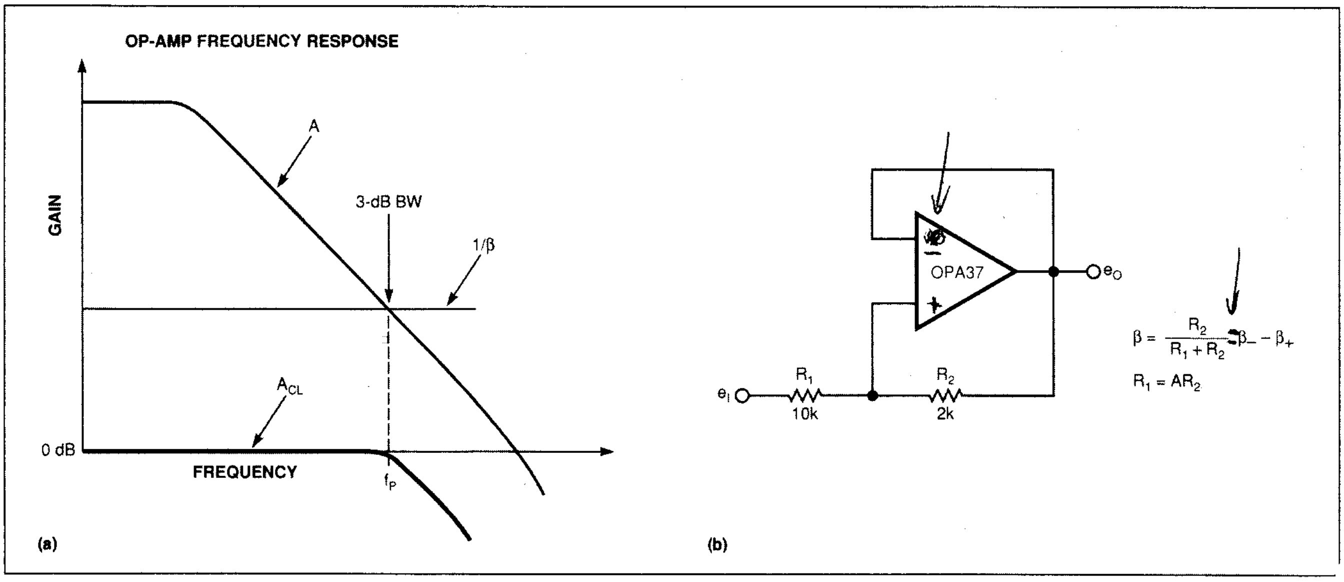

Fig 5—Feedback to both op-amp inputs *separates the 1/β and closed-loop-gain curves **(a)** for this high-slew-rate voltage follower **(b)**.*

the summation element. The summation polarity then matches that of the amplifier input the β_+ feedback-path affects. Feedback through β_- remains connected to a negative summation input, and the two feedback polarities reflect the differential nature of the amplifier. The differential inputs of an op amp cause the amplifier to respond to the difference between the signals at the two amplifier inputs. Thus, the amplifier subtracts the two feedback signals when the signals are connected to opposite amplifier inputs. The model repeats this subtraction by using opposite signs for feedback inputs to the summation element.

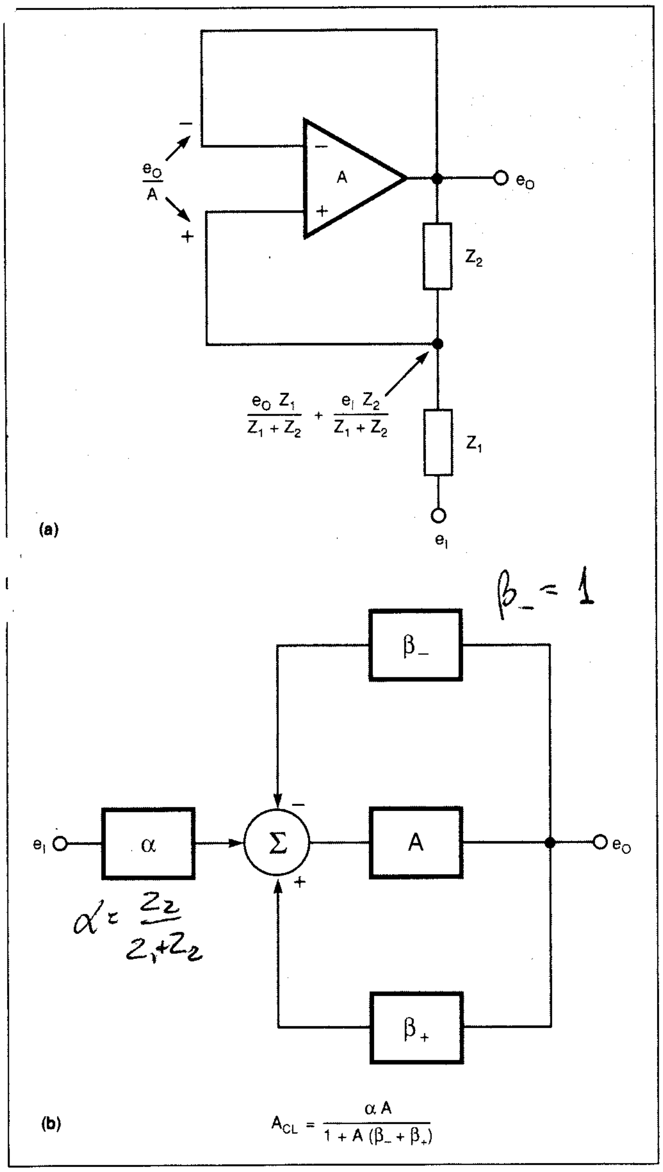

*Fig 6—**The dual feedback** of the circuit in **a** results in a model with feedback factor, β, which is equal to the difference between the negative and positive feedback factors **(b)**.*

Dual feedback subtracts feedback factors

The differential inputs' subtraction results in a net feedback factor that is the difference between the positive and negative feedback factors. Analyzing the circuit and model results in the response equations in **Fig 6**. The response denominator is of the standard form $1+(1/A\beta)$ when the net circuit feedback factor is $\beta=\beta_- - \beta_+$. Then, the equations confirm the **Fig 6** model to the amplifier configuration, and

$$A_{CL}=\frac{\alpha/\beta}{1+1/A\beta}=\frac{A_{CLI}}{1+1/A\beta}$$

$$\beta=\beta_- - \beta_+.$$

Note that, because the net feedback is negative, the net feedback factor is $\beta_- - \beta_+$ rather than $\beta_+ - \beta_-$.

To determine the ideal gain, $A_{CLI}=\alpha/\beta$, express the α and β factors in terms of circuit elements. Although the equations for **Fig 6** define these factors, depending on detailed equations is no longer necessary. Once the equations confirm the model, you do not need them to repeatedly analyze a given configuration. The A_{CL} expression of the model defines A_{CLI} in terms of factors you can determine by inspection. You determine the feedback and feed-forward factors from the associated voltage-divider ratios. The ratio is unity for the direct output-to-input connection of the **Fig 6** β_- feedback. However, the **Fig 6** model also holds for other cases in which a feedback network sets β_-.

For the specific circuit of **Fig 6**, the result is the desired voltage-follower response; however, the circuit amplifies any errors. Reading the individual factors from the **Fig 6** circuit and subtracting the two β factors gives

$$\alpha=\beta=\frac{Z_2}{Z_1+Z_2}.$$

$\beta = \beta_- - \beta_+ = 1 - \frac{Z_1}{Z_1+Z_2} = \frac{Z_2}{Z_1+Z_2}.$

Thus, $A_{CLI}=\alpha/\beta=1$ for the desired voltage-follower response. However, with $\beta<1$, the input errors of the amplifier are amplified by $1/\beta>1$. Given the β selection for **Fig 6**, $1/\beta=A_{MIN}$. Then, the input errors are amplified by approximately the same factor that slew rate is improved. For the specific components of **Fig 5**, the input errors are amplified by a factor of five, and the slew rate improves by a factor of six. The error-signal

Gain > 1, thus less compensation needed!

With feedback modeling, you can simplify op-amp circuit analysis to the determination of voltage-divider ratios.

gain rolls off in accordance with the amplifier open-loop response, as the equation for A_{CLE} (**Eq 4**) shows. You can remove the increased error gain for the input offset voltage by using a capacitor in series with R_2 in **Fig 5**.

The reduced input impedance of the **Fig 5** circuit also increases the error. However, this effect is less than you would first expect. At first, the input impedance of the circuit appears greatly reduced because the input signal drives a feedback network. Normally a voltage follower presents the very high impedance of an op-amp input to the signal source. When driving the feedback network in an inverting circuit, the input signal sees the impedance of the input resistor. This great difference in input impedance would also result for **Fig 5** except for the bootstrap action of the positive feedback. The follower action of the circuit keeps both ends of the feedback network at almost the same signal level. The only signal appearing on Z_2 in **Fig 6** is the small e_O/A. Thus, the feedback network draws very little current from the signal source. The resulting input impedance is $R_I = AZ_2$.

Dual inputs expand options

Still other op-amp configurations couple input signals to both inputs of the amplifier. For these configurations, modify the feedback model on the input rather than the feedback side. The input signals coupled to the op amp may be from the same signal source or from separate sources. In the simplest case, the same signal source supplies both op-amp inputs, serving to illustrate input modifications for the model.

Fig 7 shows the dual-coupling of a signal source to the two op-amp inputs. This circuit selectively amplifies the op amp's input-error signal for greatly improved resolution of error measurement. Distortion measurement is a prime beneficiary of this selective gain. Input-error-signal e_{ID} appears across R_1 and develops a feedback current of e_{ID}/R_1. This current also flows through R_2 and develops an amplified replica of e_{ID} at the op-amp output. The resulting error-signal gain is $(R_1+R_2)/R_1$. This gain equals $1/\beta$ as you can see by reading β from the voltage-divider ratio of the feedback network.

The amplification excludes test-signal e_I because this signal does not appear across R_1. The circuit bootstraps R_1 on top of e_I; the resistor supports only the amplifier-input-error signal. Signal e_I shifts the op-amp input voltages without developing a voltage across R_2. With no related signal on R_2, the op-amp output follows e_I. Thus, signal e_I receives only unity gain from the circuit, and the amplifier error signal receives a gain of $1/\beta$. Because of this selective amplification, the amplified error signal is far more prominent at the amplifier output. The selective gain reduces the dynamic-range requirements for the error measurement. In addition, the unity gain presented to e_I lets this signal span the full voltage range of the op-amp input without causing output saturation.

However, this selective gain also reduces the feedback factor, resulting in bandwidth reduction. Distortion measurements must accurately predict the resulting bandwidth to determine the number of higher-frequency harmonics the circuit amplifies. From a circuit perspective, the **Fig 7** configuration illustrates the effect of β on bandwidth. For this circuit, the output voltage is $e_O = e_I - (e_{ID}/\beta)$. Thus, e_O diminishes from the level of e_I in the presence of e_{ID}. Part of signal e_{ID} is the gain error, e_O/A, which causes higher-frequency roll-off in the closed-loop response. For bandwidth considerations, e_O/A replaces e_{ID}, and the resulting output voltage is $e_O = e_I - (e_O/A\beta)$. As open-loop-gain A declines with frequency, the output signal increasingly diminishes. At some point, the drop in output reaches the −3-dB point of the bandwidth limit. The circuit reaches this limit sooner because of the presence of β in the e_O equation. The roll-off effect of A is amplified by $1/\beta$, which reduces the bandwidth by the same factor. As before, $BW = \beta f_C$.

The performance of **Fig 7** is very similar to that of **Fig 5**. Both circuits maintain unity gain to the signal source but the amplifier operates with $\beta<1$. For **Fig 5**, the reduced feedback factor permits less amplifier phase compensation but results in greater gain to the error signals. **Fig 7** intentionally adds gain to the error

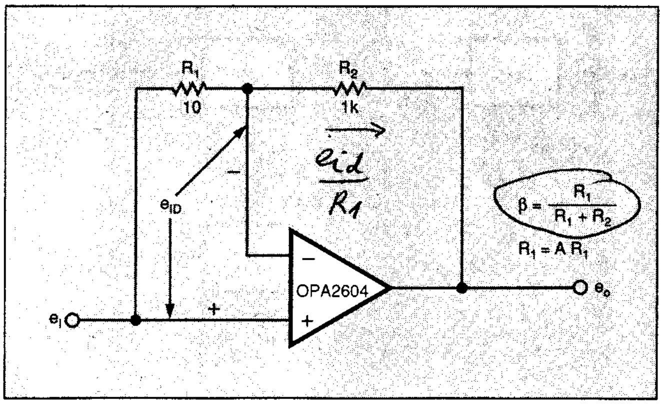

Fig 7—This circuit results in input signal coupling to both of the op-amp inputs *when R_I is boot-strapped to selectively amplify the error signal, e_{ID}.*

Using information based on the feedback factor, you can determine the frequency response and stability of an amplifier as well as its gain.

signal for measurement applications. The only difference between the two circuits lies in their applications. In practice, the circuits realize the same results through different configurations. From an applications standpoint, the two circuits are interchangeable.

The primary difference between the two circuits is in the feedback modeling. **Fig 5** demonstrates dual feedback, and **Fig 7** shows dual input connections. **Fig 8** shows the **Fig 7** modeling results by redrawing the circuit to show the two input connections. The **Fig 8** circuit couples input-signal e_I directly to the amplifier's noninverting input. For the model, the direct connection represents an α of unity and connects e_I directly to a positive input of the summation element.

The circuit also couples signal e_I, which a feedback network attenuates, to the inverting input of the amplifier. This attenuation defines a feed-forward factor equal to the voltage-divider ratio $Z_2/(Z_1+Z_2)$. In the model, the α block represents this second input connection, which goes to a negative summation input. Finally, a feedback path couples the circuit output to an amplifier input. In this path, the attenuation of the feedback network is $Z_1/(Z_1+Z_2)$, which is the feedback factor. This feedback path connects to another negative summation input in the model, which corresponds to the inverting amplifier input connection of the circuit.

Analyzing the completed model produces a transfer response of the expected form:

$$A_{CL}=\frac{(1-\alpha)/\beta}{1+1/A\beta}=\frac{A_{CLI}}{1+1/A\beta}.$$

The denominator of this equation is the $1+(1/A\beta)$ result common to all of the previous results. Thus, bandwidth and stability conclusions previously drawn from this denominator also apply to the equations for **Figs 7** and 8. The closed-loop bandwidth is βf_C, and frequency stability conditions relate to the intercept of the A and $1/\beta$ curves of **Fig 3**. The expression for the ideal closed-loop gain for the **Fig 7** and **Fig 8** circuits is the numerator of the equation, $(1-\alpha)/\beta$. Substituting the expressions for α and β in this expression shows that $A_{CLI}=1$.

Modeling extends simplicity

You can readily extend the modeling principles of the preceding examples to any op-amp application. Using this approach, the final circuit analysis reduces to a single loop equation. Moreover, feedback modeling simultaneously defines many circuit-performance characteristics while avoiding the more complex response analysis of the circuit. You analyze the actual circuit only when questions arise about the validity of the feedback model. The three steps of feedback analysis are drawing the model, determining the α and β factors, and finding the transfer response.

Drawing the model centers on the op amp's differential inputs. Feedback or input-signal connections to the op amp's inverting input are drawn as connections to negative inputs on the model summation element. Connections to the op amp's noninverting input are drawn as connections to positive summation inputs. An α or β attenuator accompanies each of these input and feed-

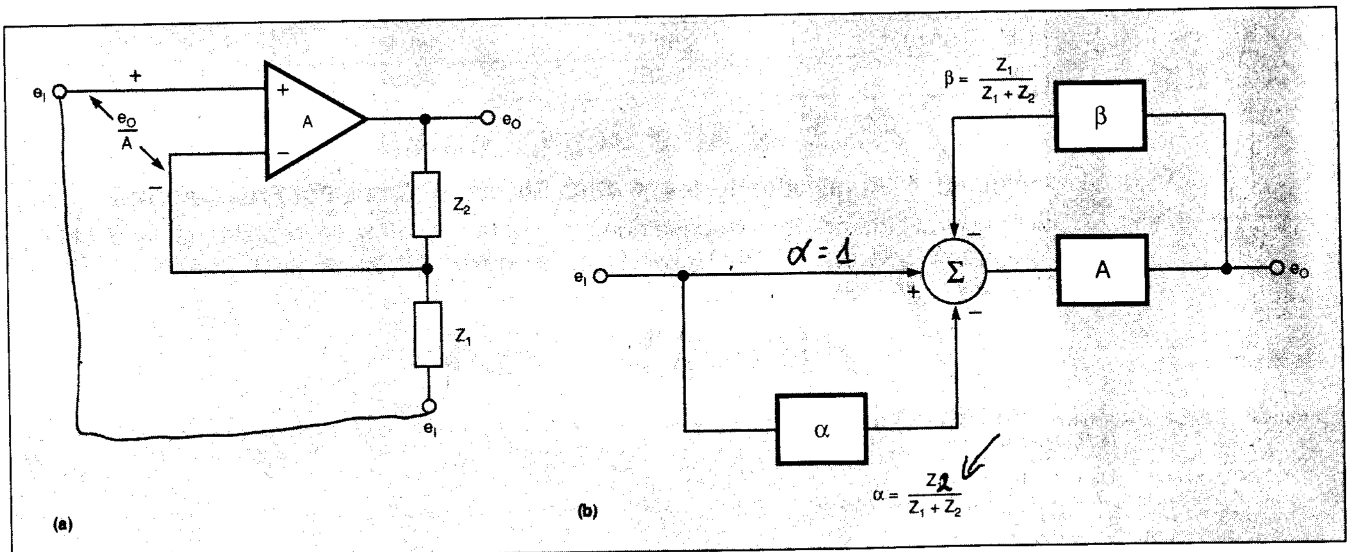

Fig 8—The direct and attenuated input connections of the circuit in ***a*** *couple to opposite-polarity inputs of the feedback model* ***(b)***.

The generalized feedback model covers each of the four possible input and feedback connections to the two op-amp inputs.

back connections. With just these polarity and attenuator guidelines, you draw the model itself. From the feedback networks, you find the individual α and β terms as voltage-divider ratios. Feedback and feed-forward signals drive a given network from opposite ends, resulting in different divider ratios. You find the two corresponding ratios by using superpositioning to separate the effects of the feedback and feed-forward signals. Once you determine these ratios, the feedback model is complete.

Next, you analyze the model to determine the net feedback-factor of the circuit and to find the ideal closed-loop gain. For most op-amp configurations, you can read the net β directly from the circuit. You read the individual β_- and β_+ factors from the voltage-divider ratios of the feedback networks. You can find the net feedback factor of the circuit from $\beta=\beta_- - \beta_+$. This step alone defines numerous performance errors as described for **Fig 1**. You can also find the bandwidth at this point through $BW=\beta f_C$. Where β varies with frequency, the value of beta used to find the bandwidth is the value at the intercept of the A and 1/β curves.

To complete the process and find A_{CLI}, analyze the model for its transfer response. This step requires one loop equation, which describes the model summation times the open-loop-gain A. Solving this equation for $A_{CL}=e_O/e_I$ defines the transfer response of the circuit in terms of A and the α and β factors. You then manipulate this result to arrange it in a standard form. The denominator of the A_{CL} result always reduces to the form $1+(1/A\beta)$, and the resulting numerator is the ideal closed-loop gain, A_{CLI}. This standard-form requirement helps you detect analysis and modeling errors.

Complex circuit yields to modeling

To illustrate feedback analysis, consider the voltage-controlled current source of **Fig 9 (Ref 3)**. Because of positive feedback, this op-amp connection produces an output current that is independent of the load voltage. The voltage load R_L develops acts as an input signal to the op amp's noninverting input. The amplification of this signal adjusts the op amp's output voltage by an amount that accommodates the load voltage. The added output voltage supplies a correction current through the positive feedback network R_2/n and R_2 form. This current accurately compensates the effect of the load voltage as long as you establish the illustrated 1:1/n resistor ratios.

The **Fig 9** circuit is well known, but its performance characteristics are not obvious. The circuit structure offers little insight into its bandwidth and the effects of input error signals. The voltage swing at the amplifier's output due to input and load voltages is not apparent. Furthermore, the circuit's positive feedback raises the question of frequency stability. Straightforward analysis of all these performance characteristics is a formidable task.

Feedback modeling reduces the task to one loop equation through the information you derive from the feedback factor and closed-loop response. **Fig 10** show the feedback-analysis circuit of **Fig 9**. This format displays positive and negative feedback factors through voltage dividers. To model the **Fig 10** circuit, you include positive and negative feedback paths around the gain block. These paths meet summation-element inputs bearing the same signs as the corresponding amplifier inputs in the circuit. The model couples input-signal e_I to the summation element through an α block, which represents the attenuation of the feedback network e_I drives.

To define the α and β terms, take the corresponding voltage-divider ratios from the circuit diagram. For the circuit of **Fig 10**,

$$\alpha=\frac{1}{1+n},\ \beta_-=\frac{n}{1+n},\ \beta_+=\frac{nR_L}{R_2+(1+n)R_L}.$$

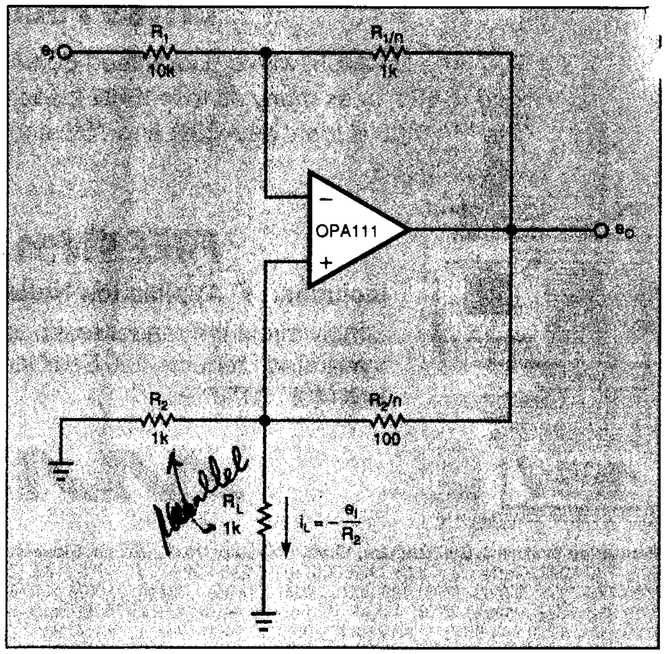

Fig 9—A complex feedback structure confuses calculation of circuit performance for this well-known current source.

Using feedback modeling, you can derive the frequency characteristics of an op-amp circuit by analyzing the model's closed-loop response equation.

You find the net circuit feedback factor from

$$\beta=\frac{R_2}{R_2+(1+n)R_L}\beta_-.$$

Generalized results define performance

With this simple analysis, you know the bandwidth, stability, and effects of amplifier errors for the **Fig 9** circuit. The resistance values yield a β of 0.076. As a result, the circuit bandwidth at βf_C is a small part of the available amplifier bandwidth. For the OPA111, f_C=2 MHz, and the circuit's bandwidth is 152 kHz. Even less bandwidth results with higher values of load resistance. As the β equation shows, the net feedback factor decreases to zero as R_L becomes very large. For **Fig 9,** an increase in load resistance from 1 to 10 kΩ reduces the circuit bandwidth from 152 to 16 kHz. Normally, the values of R_1 and R_1/n would suggest a low-gain circuit for which the bandwidth would approach that of f_C. However, an almost equal β_+ counteracts the near-unity β_-, and the resulting feedback demand for amplifier gain is high.

The frequency-stability information revealed by the β equation is twofold. First, the equation shows that β is always a positive value, indicating that negative feedback prevails regardless of the load resistance. Otherwise, the positive feedback could have dominated the circuit to cause latching or oscillation. The β equation provides further stability information through graphical analysis. Oscillation can still result if R_L is an inductive load, such as that of a motor. In this case, the load impedance rises with increasing frequency, causing a corresponding decrease in β. This decreasing β would cause the 1/β curve of **Fig 3** to rise with frequency. The increased 1/β slope signifies greater phase shift in the loop at the intercept of the 1/β and A curves. This increased feedback phase shift signifies potential response ringing or even circuit oscillation. To retain stability in these cases, bypass the load with a capacitor.

The β equations also show the effects of amplifier input errors on the **Fig 9** circuit output current. As with all op-amp configurations, the input-referred errors e_{ID} includes are first amplified by 1/β. This amplification determines the error effects at the op amp's output. From this output, the positive feedback network feeds back the errors through an attenuation factor of β_+. This attenuated signal is across load R_L and develops an output error current of $(\beta_+/\beta)(e_{ID}/R_L)$. Typically, β_+/β is large, and the effects of e_{ID} are amplified in the load current. For the components of **Fig 9,** $\beta_+/\beta=11$, which is the gain the circuit applies to the errors of e_{ID} before those errors appear across R_L.

Continuing the model analysis yields the **Fig 9** transfer response. You derive the current-output response from the input-to-output voltage response, e_O/e_I. Using

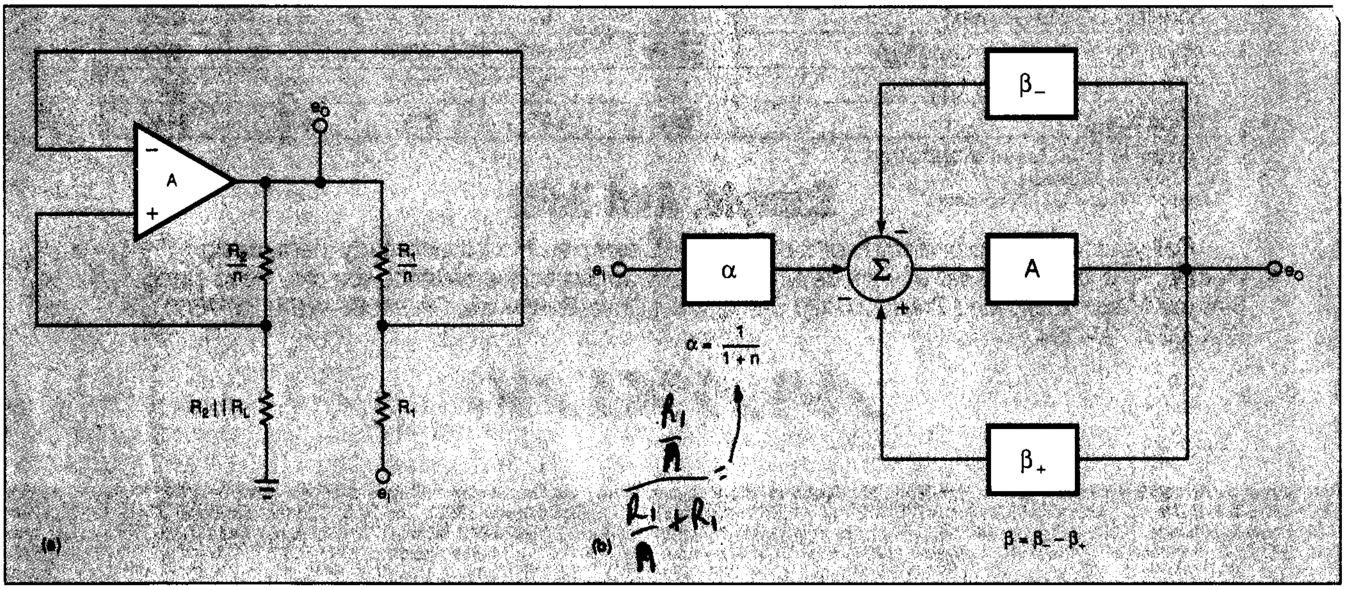

***Fig 10—Translating the Fig 9 circuit** into a feedback-analysis circuit (**a**) and then a feedback model (**b**) simplifies analysis and extend performance insight through standardized feedback results.*

Frequency plots let you evaluate the frequency stability of an op-amp circuit from the A and 1/β curve slopes.

the **Fig 10** model, you find e_O/e_I from a single loop equation that you then reduce to standard form. From the model, $e_O = A(-\alpha e_I - \beta_- e_O + \beta_+ e_O)$. Solve this expression for $A_{CL} = e_O/e_I$ and manipulate the result to develop the standard denominator of $1 + (1/A\beta)$. For **Fig 10,**

$$A_{CL} = \frac{e_O}{e_I} = \frac{-\alpha/\beta}{1 + 1/A\beta}.$$

You then translate this result to a current output by first noting that the load voltage equals the positive feedback signal, or $e_L = \beta_+ e_O = i_L R_L$. Solving this equation for e_O and substituting the result in the A_{CL} equation yields a **Fig 9** response of

$$\frac{i_L}{e_I} = \frac{-1/R_2}{1 + 1/A\beta}.$$

Generalized model covers all

Drawing and analyzing feedback models adds insight to op-amp circuit operation and works with any op-amp application. However, op-amp circuit analysis is even simpler with a generalized feedback model and standard response equations. These standardized results avoid even the single loop equation of the model analysis and hold for all practical applications. Op-amp circuit analysis then reduces to finding voltage-divider ratios, which you can generally determine by inspection.

The feedback model of **Fig 11** represents all possible op-amp circuit configurations. This model includes input and feedback connections to both the positive and negative summation inputs. The separation between the possible and the practical excludes op-amp configurations that have no end value.

The **Fig 11** model represents each of the four possible input and feedback connections to the two op-amp inputs. Most op-amp configurations do not use all of these connections. In these cases, you set the associated α or β terms to zero. Similarly, many op-amp applications have direct input or feedback connections to the op-amp inputs. In these cases, a network does not attenuate the related signals, and you set the associated α and β terms to unity. For example, the **Fig 10** circuit has no input-signal coupling to the op amp's noninverting input. This lack of input-signal coupling sets α_+ to zero, and the **Fig 11** model reduces to the model in **Fig 10**. Similarly, the **Fig 8** circuit has no feedback

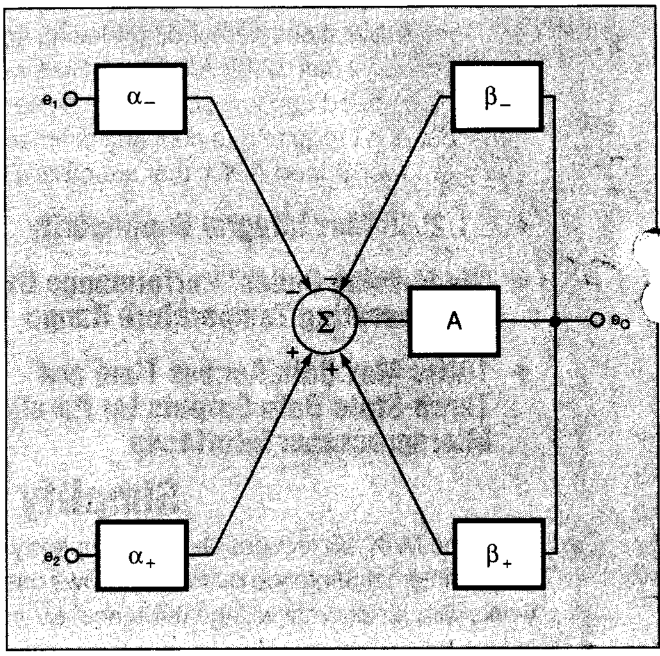

Fig 11—A generalized feedback model and standardized response equations *reduce op-amp circuit analysis to finding α and β voltage-divider ratios.*

coupling to the op amp's noninverting input, and the input signal connects directly to this input. In this case, $\beta_+ = 0$, $\alpha_+ = 1$, and the generalized model reduces to the model in **Fig 8**.

Analyzing the generalized model yields standardized equations that also lend themselves to specific op-amp applications. For the model of **Fig 11**,

$$A_{CL} = \frac{e_O}{e_I} = \frac{\alpha/\beta}{1 + 1/A\beta} = \frac{A_{CLI}}{1 + 1/A\beta} \quad (5)$$

$$\beta = \beta_- - \beta_+.$$

This analysis immediately communicates three results common to all op-amp configurations. First, the net feedback factor of the circuit is $\beta = \beta_- - \beta_+$. In all cases, the differential inputs of the op amp subtract one feedback signal from the other. Next, the denominator of the A_{CL} equation is the familiar $1 + (1/A\beta)$. Thus, the bandwidth and frequency-stability conclusions drawn using this denominator still apply. As in **Fig 3**, the intercept of the 1/β and A curves sets the **Fig 11** bandwidth $BW = \beta f_C$. Frequency stability relates to the curve slopes at this intercept, as also described for **Fig 3**. Finally, the numerators of **Eq 5** show that the ideal closed-loop gain is $A_{CLI} = \alpha/\beta$ regardless of the op-amp configuration. Because of the denominator form of **Eq**

You can write the transfer response of any negative-feedback op-amp system in a form that includes the $1+1/(A\beta)$ denominator.

5, A_{CL} reduces to the numerator term when loop-gain $A\beta$ is large. A large $A\beta$ again denotes the ideal region of operation.

Two variables in the generalized A_{CL} equation are defined differently for different op-amp configurations. This flexibility permits one standard response equation for all configurations. In the A_{CL} equation, both α and e_I depend on the input connections of the specific circuit. The varied participation of the modeled α_- and α_+ determine feed-forward factor α. One or the other of these α terms applies to most op-amp connections; other connections involve both terms. Input signal connections dictate the relevant α terms. α_- attenuates signals connected to the e_1 terminal of the model; the signals then go to a negative summation input. For signals at the e_2 input, α_+ is the attenuator, and the summation input has a positive polarity.

The input signal, e_I, in the standard equation accommodates this α variability; the modeled input signals are e_1 and e_2. This generalized e_I signal permits various combinations of signals e_1 and e_2. You can model the input signal connections using one equation for inverting, noninverting, differential, and common-mode cases. For each of these cases, a corresponding α term results as summarized in the following table:

Cases	e_I	α
Inverting	e_1	$-\alpha_-$
Noninverting	e_2	α_+
Differential	e_2-e_1	$\alpha_+=\alpha_-$
Common mode	$e_1=e_2$	$\alpha_+-\alpha_-$

Examining this table shows agreement with previous modeling results. From the table, an inverting input connection of e_1 couples through α_- to a negative input. Hence, the table's α term and polarity. This is the input case for **Fig 10**, which has a $-\alpha=-\alpha_-$ term in the response equation's numerator. For **Fig 6**, the input signal couples to a noninverting amplifier input for a $\alpha=\alpha_+$ term in the numerator of **Eq 5.** Similarly, **Fig 8** shows a common-mode input case, and the resulting response numerator has a factor of $1-\alpha$, which is $\alpha_+-\alpha_-$. For the differential-input case, the table shows that **Eq 5** accepts $e_I=e_2-e_1$ for $\alpha=\alpha_+=\alpha_-$.

One additional standard equation applies when using the generalized model. The equation for error-signal gain is

$$A_{CLE}=\frac{1/\beta}{1+1/A\beta}.$$

α=1 because eid is directly connected to input.

For the model, consider an input error source, $-e_{ID}$, directly coupled to a positive summation input. This addition indicates that the amplifier input errors are in series with the input circuit. With the e_{ID} source connected to the model, analysis shows that **Fig 11** amplifies input-referred errors by the same gain described earlier. EDN

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Author's biography

Jerald Graeme has been with Burr-Brown for 25 years and is the manager of instrumentation components design. Jerry has developed numerous linear ICs including op amps, instrumentation amplifiers, analog multipliers, V/F converters, and D/A converters. He has a BSEE from the University of Arizona, and a MSEE from Stanford University. In his leisure time, he enjoys scuba diving, photography, and woodworking.

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