Generalized op-amp model simplifies analysis of complex feedback schemes

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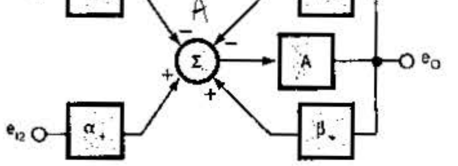
When attempting to find a path through a forest of complex op-amp feedback arrangements, you can lose your way amidst all the branches on the trees. Consolidating the branches into standard blocks for a generalized op-amp model can help you forge a simple analysis path.

Feedback configurations for op amps having multiple inputs can be as convoluted as a labyrinth. Many complex feedback schemes don't conform to simple models and often pose difficult analysis challenges. A generalized op-amp model based on voltage-divider ratios can standardize analysis to cover a variety of feedback and input configurations. By making your circuit conform to this model, you can systematically determine its gain, bandwidth, frequency stability, and errors. Three circuits that illustrate the model's utility are a modified difference amplifier, an op amp with a multiplier in its feedback loop, and an op amp with a multiplier in its feedforward path—a voltage-controlled lowpass filter.



e_I	α
e_1	$-\alpha_-$
e_2	α_+
$e_2 - e_1$	$\alpha_+ = \alpha_-$
$e_1 = e_2$	$\alpha_+ - \alpha_-$

Fig 1—A generalized model consolidates and directs all feedback and feedforward elements for a dual-input op amp towards a single summing junction.

Fig 1 shows the generalized model. The model represents the op amp and all possible feedback and input connections to it. It consolidates the sum and difference operations associated with the op amp's two inputs, e_{I1} and e_{I2}, into a single summing junction, Σ. (Note that the net op-amp input is e_I.) Composite elements for positive and negative feedback are consolidated in blocks β_+ and β_-, respectively. Similarly, the composite elements that represent the attenuation for positive and negative inputs to the op amp are consolidated in blocks α_+ and α_-. Block A represents the idealized op amp's gain, and e_O is the op amp's output.

The closed-loop gain (A_{CL}) of the generalized model is

$$A_{CL} = \frac{e_O}{e_I} = \frac{\alpha/\beta}{1+1/A\beta} = \frac{A_{CLi}}{1+1/A\beta}, \qquad (1)$$

where $\beta = \beta_- - \beta_+$ and ~~is~~ $= \alpha_- - \alpha_+$ the net amount of feedback for the model, and α and is the net feedforward attenuation for the inputs to the op amp (see table in **Fig 1**). The numerator α/β is the ideal closed-loop gain (A_{CLi}) when the open-loop gain ($A\beta$) is much larger than 1. The denominator of the equation, $1+1/A\beta$, is an expression that lets you determine the circuit's closed-loop bandwidth.

The model's closed-loop bandwidth is $BW = \beta f_C$, where f_C is the unity-gain crossover frequency of the op amp's forward gain (A). The generalized model lets you determine the gain for any error signals present at the op amp's input terminals. All op amps amplify these error signals by a gain of

$$A_{CLe} = \frac{1/\beta}{1+1/A\beta}.$$

You can apply this closed-loop error gain to any of the op amp's error signals, such as the input-offset voltage, input-noise voltage, and common-mode-rejection voltage.

The **Fig 1** model can accommodate one input to the op amp's positive input via the α_+ block and one input to the negative input via the α_- block. For multiple inputs, you must add α blocks. You determine the transfer function for each α block using superposition principles.

To illustrate the utility of the generalized model, we'll analyze some op-amp circuits having complex feedback and input configurations. **Fig 2a** depicts a configuration where the gain of an added op amp (IC_2) modifies the feedback of difference amplifier IC_1. Without the added op amp, the equal-valued resistors (R) set the difference-amplifier gain to 1. The fixed resistor values maximize the common-mode rejection ratio, and factory adjustments can trim the resistor ratios to within 0.002%.

IC_2 lets you change the circuit gain without affecting the difference amplifier's common-mode rejection ratio. You could analyze this circuit using A, α, β, and Σ blocks for each amplifier. Using the generalized model, however, you can simplify analysis by including IC_2 in the β_+ feedback path for IC_1. The fraction of the output voltage (e_O) fed back to IC_1's noninverting input is $\beta_+ = -\frac{1}{2}R_2/R_1$. Because there is no feedback to IC_1's inverting input, $\beta_- = 0$. The net feedback factor is $\beta = \beta_- - \beta_+ = R_2/2R_1$.

The equal values for the resistors surrounding the difference amplifier determine $\alpha_- = \alpha_+ = \frac{1}{2}$. In addition, the differential input signal is $e_I = e_{I2} - e_{I1}$. Substituting e_I, α, and β into the equation $A_{CLi} = e_O/e_I = \alpha/\beta$ yields

$$e_{OI} = \frac{R_1}{R_2}(e_{I2} - e_{I1}),$$

where e_{OI} is the ideal output signal. For the resistor values in **Fig 2a**, the addition of op amp IC_2 modifies the difference amplifier's ideal closed-loop gain (A_{CLi}) from 1 to 10.

You can analyze the circuit's frequency stability by examining the frequency-response curves for IC_1's open-loop gain, $|A_1|$, and the inverse of the net feedback factor, $1/\beta$, (**Fig 2b**). The two curves intersect at frequency BW. For **Fig 2b**, the rate of closure—the difference in the slopes of the $|A_1|$ and $1/\beta$ curves where they intersect—is 20 dB/decade. This rate implies a 90° phase margin and excellent overall frequency stability.

The unity-gain crossover frequency for IC_2 is f_{C2}, and IC_2's net feedback factor is $\beta_2 = R_1/(R_1 + R_2)$. The rise in the circuit's $1/\beta$ curve at $\beta_2 f_{C2}$ shows the effect of IC_2's closed-loop bandwidth on stability. When $\beta_2 f_{C2} \geq 10BW$, the rise in the $1/\beta$ curve has negligible effect on stability. The typical crossover frequencies for the INA105 (IC_1) and OPA627 (IC_2) amplifiers are 2 and 16 MHz, respectively. Using $\beta = 0.05$ and $\beta_2 = 0.91$, $BW = \beta f_{C1} = 100$ kHz, and $\beta_2 f_{C2} = 14.6$ MHz. The effect of IC_2's closed-loop bandwidth on stability is negligible.

Two op amps can be better than one

You can also readily analyze the **composite amplifier combination in Fig 3a** using the generalized model. In this example, $A_{CL2} = (1 + R_4/R_3)$ is the closed-loop gain of IC_2, and A_1 is the open-loop gain of IC_1. The second amplifier modifies gain-block A of the generalized model so that $A = A_1 A_{CL2}$. The transfer function for the A block has two poles (**Fig 3b**). One pole is due to the dominant pole for amplifier IC_1; the other pole is due to the closed-loop response of IC_2—$BW_2 = \beta_2 f_{C2}$. For frequency stability, the BW_2 pole must

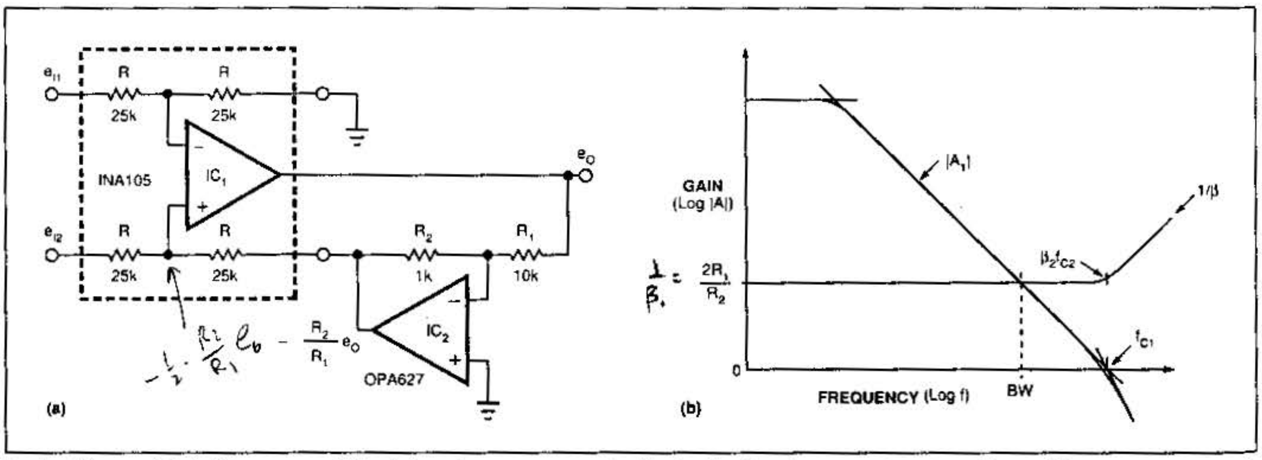


Fig 2—This modified difference amplifier (a) employs an op amp in the feedback to increase the gain without affecting common-mode rejection. For stability, the 1/β and |A| gain curves should intersect at a rate of closure of 20 dB/decade (b).

be considerably higher in frequency than the intercept point of the $|A_1|$ response curve and the $1/\beta$ response curve. For the circuit, $1/\beta = 1 + R_2/R_1$.

Introducing the second op amp reduces the error signals and increases the gain-bandwidth product of the circuit. The increase in loop gain due to IC_2 proportionally reduces the output error signal, $e_{Oe}/A\beta$, caused by any error signal at IC_1's input terminals. The loop gain is the gain difference between the $|A|$ and $1/\beta$ curves. In addition, adding IC_2 increases the circuit's bandwidth from BW_1 to BW_2. For the resistor values in **Fig 3a**, the bandwidth increases by a factor of 10.

The generalized op-amp model also helps you analyze circuits having variable feedback elements. Variable feedback elements can include potentiometers, switches, and analog multipliers, all of which affect the magnitude and phase responses for $1/\beta$. The circuit in **Fig 4a** places an analog multiplier in the feedback loop of an op amp to create an analog divider. The divider's transfer function is $e_O = 10R_2e_I/R_1e_C$. To employ the generalized model, put the gain of the multiplier in the β block. Without the multiplier, the feedback factor for op amp IC_1 is $R_1/(R_1 + R_2)$. The multiplier's transfer function, XY/10, scales e_O by $e_C/10$. Therefore, the net feedback factor for the model, which takes into account e_C, is

$$\beta = \frac{e_C R_1}{10(R_1 + R_2)}.$$

Fig 4b illustrates the effect of the e_C variable control signal on the circuit's bandwidth and stability. The control-signal voltage moves the intercept point for the $1/\beta$ curve over the full range of the $|A|$ curve. When e_C is 0V, β is zero, and the op amp operates open loop. When e_C is 10V, the multiplier gain is 1,

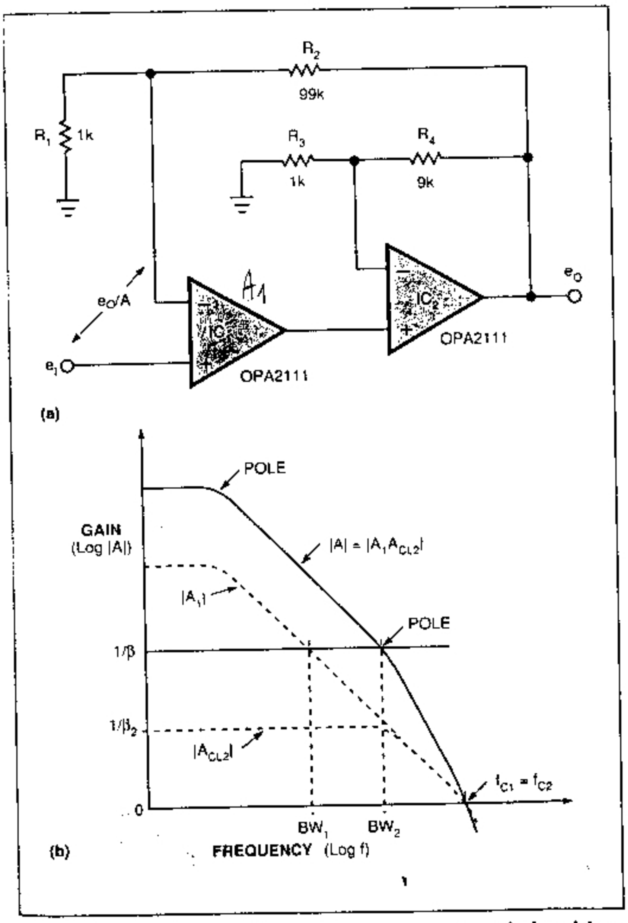


Fig 3—You can increase the net open-loop gain and closed-loop bandwidth by placing an additional op amp in the forward gain path (a). The closed-loop response of the additional amplifier places a second pole in the A gain block (b), which determines the circuit's stability.

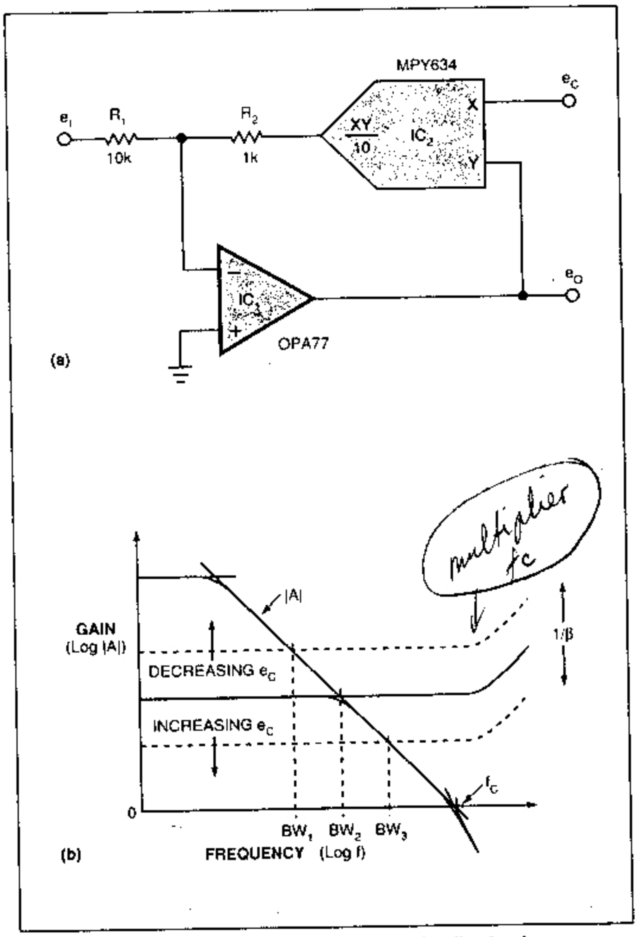


Fig 4—The addition of a multiplier in the feedback of an op amp lets you divide inputs from two signal sources (a). The net feedback factor β varies with control voltage e_C (b), so you must analyze the circuit's stability over the entire control-voltage range.

and the device can be replaced with a short circuit. Because the intercept point varies, you must pay close attention to the rate of closure of the two curves over the entire control-voltage range.

A pole in the multiplier's frequency response causes the $1/\beta$ curve to rise, as Fig 4b shows. To maintain circuit stability, the op amp must have a dominant pole so the moving intercept point always occurs on the flat portion of the $1/\beta$ curve. The OPA77 op amp's 0.6-MHz bandwidth and the MPY634 multiplier's 16-MHz bandwidth make the devices good choices to ensure the circuit's frequency stability. For the component values in **Fig 4a**, β varies from 0.00091 to 0.091 as e_C varies from 0.1V to 10V. The circuit bandwidth changes proportionally from 550 Hz to 55 kHz.

To use the generalized model for circuits having multiple inputs to the op amp, you must apply superposition principles. **Fig 5a** shows an example where three input signals connect to the inverting input of an op amp through different networks. **Figs 5b** and **5c** show redrawn versions of the circuit that let you use superposition to determine α and β, respectively, for the generalized model. To determine β, use e_O as a signal source, and set all of the input sources to ground. Under these conditions, the net feedback factor is

$$\beta = \frac{R_1 \| R_2 \| R_3}{R_1 \| R_2 \| R_3 + R_F}.$$

For the resistor values in **Fig 5a**, $\beta = 0.009$.

To determine α for a particular input, set all of the signal sources except the input under consideration to ground. To determine α_1 for input-source e_1, calculate the voltage divider to the op amp's inverting input when e_2, e_3, and e_O are set at ground:

$$\alpha_1 = \frac{R_F \| R_2 \| R_3}{R_F \| R_2 \| R_3 + R_1}.$$

For the resistor values in **Fig 5a**, $\alpha_1 = 0.9$. Using the same approach to calculate the αs for the input sources at e_2 and e_3 yields $\alpha_2 = 0.09$ and $\alpha_3 = 0.009$.

Although the attenuation (α) block for each input source is different, the circuit's bandwidth and stability considerations are the same for all input sources. The reason is that the denominator for A_{CL} in **Eq 1**, which is $1 + 1/A\beta$, is the same for all input sources and doesn't depend on α. The circuit's bandwidth is $BW = \beta f_C$ for all input sources.

Increasing the number of op-amp inputs constricts the bandwidth of the circuit, however. For the **Fig 5** example, if e_3 is the only input source, the net feedback factor is $\beta = R_3/(R_3 + R_F) = 100\ k\Omega/(100\ k\Omega + 100\ k\Omega) = 0.5$. The OPA111 has a unity-gain cross-over frequency (f_C) of 2 MHz, so the circuit bandwidth is $BW = 0.5 \times 2\ MHz = 1\ MHz$. The addition of the other two input sources to the circuit reduces the net feedback factor to $\beta = 0.009$. Therefore, the added input sources decrease the circuit bandwidth to 18 kHz—a factor of 55.

The different α blocks determine different ideal closed-loop gains for each input source. The ideal closed-loop gain expression is $A_{CLi} = \alpha/\beta$. Therefore, the ideal closed-loop gain for the input sources at e_1, e_2, and e_3 are -100, -10, and -1, respectively. However, the closed-loop bandwidth is still 18 kHz for all three input sources.

Finally, look at an op-amp configuration that employs multiple feedback paths. **Fig 6** depicts a circuit that has feedback from the op amp's output and from the output of an added multiplier. The multiplier is a gain block in series with the circuit's forward gain path and alters the net forward gain as well as the net feedback factor. The multiplier and the resistor feedback convert the circuit from an integrator to a voltage-controlled lowpass filter.

The multiplier's transfer function is XY/10. When

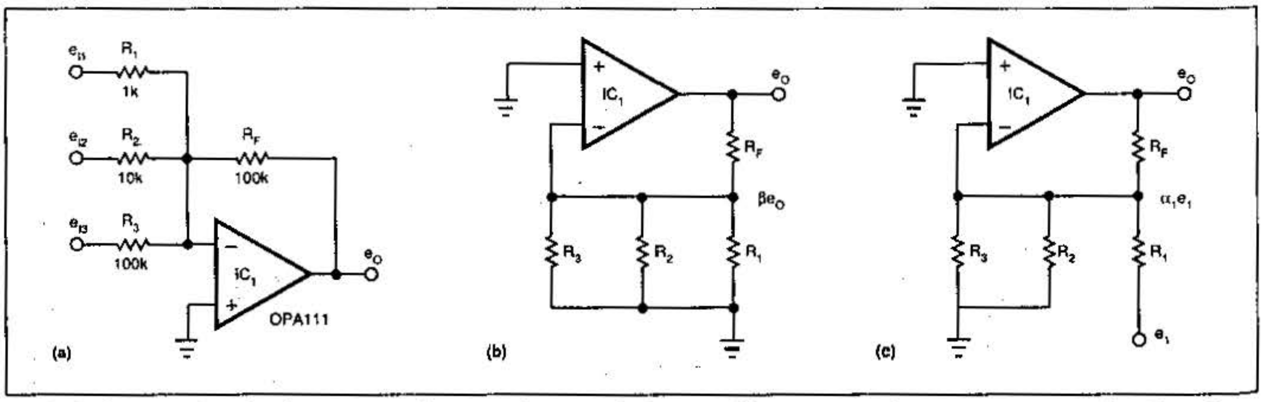


Fig 5—To determine β and α for op-amp configurations having multiple inputs (a), you must apply superposition principles. The redrawn models let you determine β (b) and α (c) using parallel equivalent resistors for the respective voltage dividers.

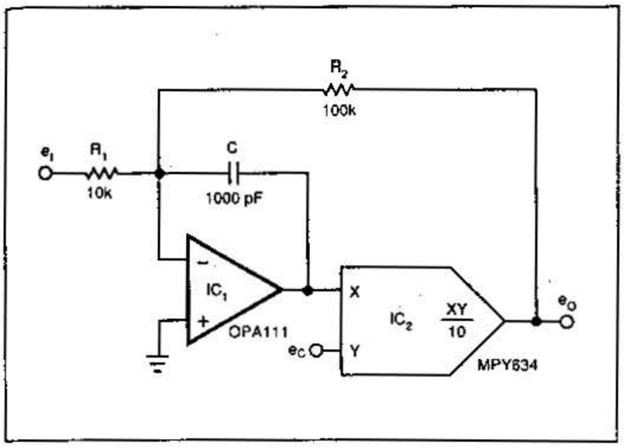


Fig 6—Adding a multiplier in the feedforward path changes a basic integrator circuit into a voltage-controlled lowpass filter. This circuit shows how you can use the generalized model to analyze an op-amp configuration that employs multiple feedback paths.

control-voltage e_C is 10V, the multiplier acts as a short circuit. Under this condition, resistor R_2 and capacitor C feedback produce a standard lowpass filter having a pole frequency, f_P, at $1/2\pi R_2C$. When e_C is less than 10V, the attenuation to the voltage at pin X on the multiplier is $e_C/10$. The attenuation reduces the voltage driving R_2, which in turn reduces the current feedback to the op amp's summing junction. The effect is the same as increasing R_2's value to $10R_2/e_C$. The increase in the effective resistance changes the lowpass-filter pole frequency to $e_C/20\pi R_2C$.

To analyze this circuit's performance using **Eq 1** for the generalized model, you must determine A_{CLi}, A, and β. At very low frequencies, the closed-loop gain is $-R_2/R_1$, because the capacitor has no effect. At higher frequencies, the voltage-controlled corner frequency determines the closed-loop roll-off. Therefore, the closed-loop gain is

$$A_{CL}=\frac{e_O}{e_I}=\frac{A_{CLi}}{1+1/A\beta}=\frac{-R_2/R_1}{1+s(10R_2/e_C)C}.$$

Both the op amp and the multiplier determine the forward gain of the model, so $A=A_1e_C/10$. The net feedback factor, β, depends on two feedback paths. **Fig 7a** depicts the feedback model for the **Fig 6** circuit. The β_C and β_R blocks represent the feedback to the op amp's inverting input through the capacitor and resistor (R_2), respectively. The net feedback factor is

$$\beta=(10/e_C)\beta_C+\beta_R.$$

The voltage fed back through the capacitor precedes the attenuation caused by the multiplier. Therefore, β_C is magnified by $10/e_C$, because the voltage at the multiplier's input is $(10/e_C)e_O$. Fig 7b shows the 1/β and | A | curves for the Fig 6 circuit. The dashed curves show how the two curves change with variations in e_C. The 1/β and | A | curves move up and down

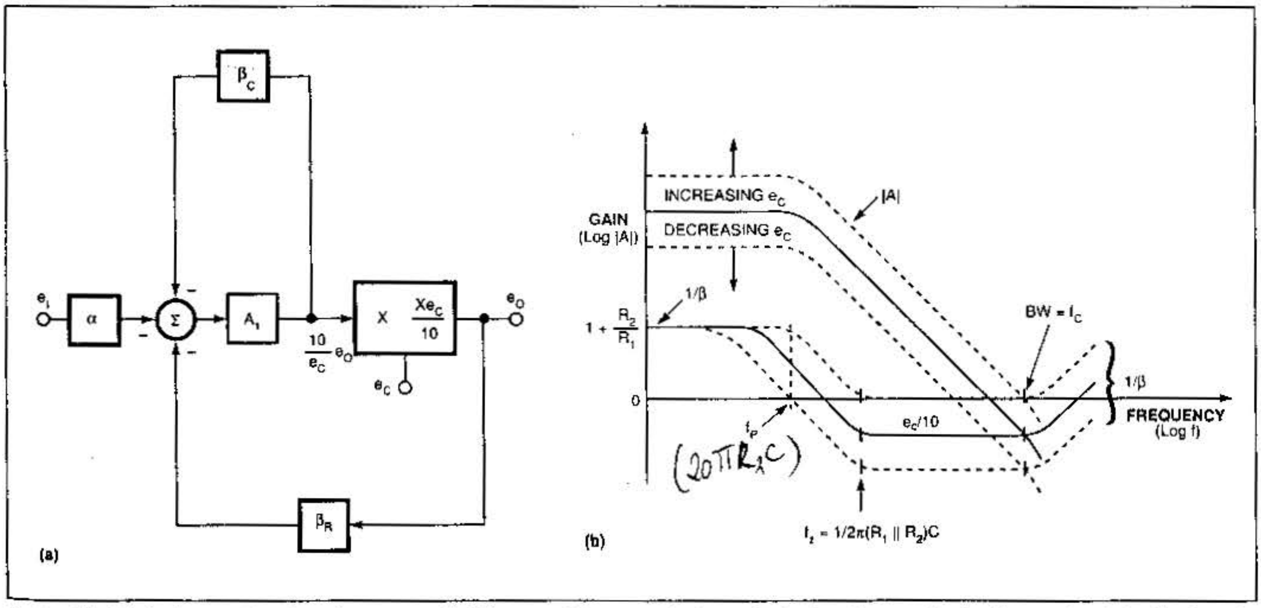


Fig 7—The block diagram for the voltage-controlled lowpass filter reveals the multiplier's effect on the feedforward and feedback paths (a). When analyzing the circuit's stability, you must consider the variations in the |A| and 1/β curves over the entire control-voltage range (b).

together, so the intercept point and corresponding bandwidth remain fixed. The rise in the 1/β curve at high frequencies is due to the bandwidth of the multiplier and determines the rate of closure for the two curves.

The circuit demonstrates a condition not usually found in op-amp configurations. When the multiplier provides attenuation, the feedback is greater than 1 at high frequencies. Generally, the feedback cannot exceed 1, but in this case the multiplier attenuation causes a signal larger than the e_O output signal to feedback through the capacitor. Therefore, the 1/β curve drops below the unity-gain axis. The circuit remains stable, however, because the intercept point is fixed.

The shape of the 1/β curves indicate the frequencies at which either of the two feedback paths dominate. Feedback is primarily through the β_R block in the flat portion of the 1/β curve labeled $1+R_2/R_1$. Feedback through the β_C block begins at the pole located at $f_P=20\pi R_2C$ and causes the slope of 1/β curve to decrease at 6 dB/octave. Feedback is primarily through the β_C block in the flat portion of the 1/β curve labeled $e_C/10$. The curve before this flat portion levels off at the zero located at $f_Z=1/2\pi(R_1 \parallel R_2)C$, where $R_1 \parallel R_2$ is the equivalent parallel resistance of R_1 and R_2. EDN

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Author's biography

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